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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

FLOW OF WATER IN TIDAL CANALS

By Earl I. Brown,* M. Am. Soc. C. E.

Synopsis

The passage of the ocean tidal wave before the mouth of an estuary, river, or inlet sets up therein a derived wave, which is propagated into the mouth in accordance with certain laws not yet clearly understood by engineers. The simplest example of a channel in which a derived wave may be propagated is a canal of uniform width and depth. This paper is devoted to a consideration and further study of the phenomena resulting from the propagation of derived waves in inland channels.

The derived wave is considered to be a wave of translation with successive alternating positive and negative phases, propagated in accordance with known laws. Proper allowances are made for the effects of friction on the bottom and sides of the channel. The problem is treated from the standpoint of the expenditure of the energy imparted to the derived wave by the generating wave in overcoming this friction.

The length, width, and depth of the canal have an effect in determining the rate at which this energy is expended. The conditions under which the derived wave is propagated are also modified by the character of the body of water with which the canal communicates at its farther end. Three cases are considered, namely, (*a*) a body of water at constant level; (*b*) a body of water with varying level, but with no proper tide of its own; that is, one that depends for its variation on the wave propagated through the canal; and (*c*) a body with varying level, but having a tide of its own derived from some source independent of the tide propagated through the canal.

Theoretical considerations are discussed, first, in order to enable fundamental conceptions to be established as to the mechanism of propagation, and as to the influence of width, depth, and canal outlet on propagation. These fundamental conceptions lead to the proposal of what may be termed the "Reflected Wave Theory" to account for the effect on the derived wave of the characteristics of the body of water at the outlet of the canal. This theory is believed

Note.—Written discussion on this paper will be closed in **April, 1931**, *Proceedings*.

* Col., Corps of Engrs., U. S. A.; Dist. Engr., U. S. Engineer Office, Philadelphia, Pa.

to be new; the writer has not heretofore encountered it in his researches in literature on tidal subjects.

As they usually exist, no canals are so long that friction will absorb all the energy of the derived wave. Therefore, the presence of the unexpended energy of the wave, when it arrives at the end, or the outlet, must be explained. According to the reflected wave theory this remaining energy is propagated back into the canal in the form of a reflected wave the energy of which is also subject to frictional losses; if the remaining energy is not entirely consumed in the course of the transit of the reflected wave, a second reflected wave is set up, and so on indefinitely until practically all the energy of the derived wave shall have been absorbed by friction within the limits of the canal.

If the canal outlet is restricted or closed, the reflected wave will be of the same sign with respect to height as the derived wave. If the outlet is wider than the width of the canal, it will be of the opposite sign.

These various waves, derived and reflected, exist simultaneously in the canal, and accordingly they will interfere at all points in accordance with the laws of the interference of simultaneous waves. The derived wave as actually observed in the channel is the resultant formed from this interference.

Wave energy is manifested both in the height of the wave above or below mean sea level and in the velocity of the water set up by the wave propagation. In the simple wave of translation, the height of the wave and the accompanying velocity of the water are roughly proportional. When two simple waves of translation interfere while being propagated in opposite directions, the heights will be added to form the resultant height, and the velocities will be subtracted to form the resultant velocity, since the two component velocities are traveling in opposite directions. Hence, in the resultant, the height and velocity are no longer proportional to each other.

While the longitudinal profile of the generating wave is not usually truly sinusoidal, it may be so considered without appreciable error; hence, the derived wave, including the resultant and all reflected waves, may be considered as sinusoidal. This approximation makes it unnecessary to compute the effect of friction in reducing each element of the derived wave, but only of the summit of the wave. All other elements follow from that by an application of the cosine law.

To apply this theory, an equation is first deduced by which the height of the wave summit, as reduced by friction, may be found for any point in the canal. An extension of this computation by the same formula gives the height at the selected point of as many reflected waves as have sufficient magnitude to warrant their consideration. The cosine law is applied to each of these with proper allowances for phase differences resulting from the various times required for each wave to be propagated from the entrance to the selected point, and the summation of these results gives the resulting wave height. A similar process gives the resulting velocity at the same point.

The method of applying the theory to canals of various characteristics is shown, and methods of determining the coefficient of friction for each case are indicated.

Lastly, the theory is tested by making the computations in accordance with it for certain known canals and then comparing the computed results with the results of observation.

Definitions

The term, "tide," relates to the rise and fall of the water due to the propagation of a derived wave.

The movement of a "simple wave of translation" causes a particle of water to remain permanently displaced in one direction or the other. This wave is usually referred to under the same "wave of translation", in which the height of the surface and the velocity of the water at a given point are approximately proportional.

A "compound" or "resultant wave of translation" is formed by the interference of two or more simultaneous simple waves of translation.

The "speed" of a wave is the rate at which its form is propagated. It is not to be confused with the velocity of the water.

"Mean sea level" is that at which the body of water in which the generating tide is propagated would stand if there were no tidal action in it. It is the plane of reference used herein.

"Mean canal", or "mean river level", is the mean of the high and low-water levels at any point.

The "depth" of the channel referred to herein is the reduced depth. It is the ratio of the area of cross-section referred to mean sea level to the mean width at the same level.

"High-water stand," or "low-water stand" refers to the instant at which the vertical tidal variation changes from rising to falling, or *vice versa.*

The "summit" of a wave is that element at its greatest rise or fall.

The "velocity" is the rate of motion of a particle of water acted on by a wave of translation. Velocity due to other causes is neglected in this study.

The "strength of the tide" is its maximum mean velocity.

"Slack water" refers to the instant at which the mean velocity in any cross-section is zero.

The "turn of the current" is the instant at which the current at any point changes direction. It usually occurs at or near the instant of slack water.

The "local tidal curve" expresses the relation between height of tide and time at any point in the canal.

The "instantaneous tidal curve" indicates the height of the tide at all points in the canal at a given instant.

The "geometric loci of high and low water" are the curves enveloping the local tidal curves.

The "local velocity curve" expresses the relation between the velocity and time at any point in the canal.

The "instantaneous velocity curve" indicates the velocity at all points in the canal at a given time.

The "geometric loci of slack waters" are the curves indicating the height of tide at which high or low-water slack occurs.

Notation

C = speed at which the wave is propagated; C' denotes the rate at which a wave is propagated between two given points.

c = time before the instant, $t = 0$, when the tide was at mean sea level; that is, when $t + c = 0$.

D = reduced depth of the canal.

d = difference in times between the arrival of high water at the ends of the canal.

E = energy of any unit length of a wave.

e = elevation of mean sea level corresponding to the staff reading, R.

F' = work done by frictional resistance.

F = friction per square foot of the water on the wetted surface of the bed of the canal.

f_0 = coefficient of friction depending on the nature of the bottom, the water depth, and the velocity of flow.

g = acceleration due to gravity.

h = height of the tide referred to mean sea level at any point, x, corresponding to any time, t.

H = height of the tide at the summit of the wave (Point x), at the time, t_m.

H_0 = maximum height of the generating wave (and, consequently, of the derived wave) at the canal entrance; H_1, H_2, H_3, etc., denote the value to which H_0 becomes reduced by friction in the first, second, third, etc., passages of the simple derived wave of translation. They each have different values for each value of x.

h_v = velocity height, or the height of a simple wave of translation which, by itself, would create a velocity, V.

K = a constant.

l = width of the canal at sea level.

L = length of a wave.

M = surface area of a reservoir.

p = wetted perimeter of any cross-section at mean sea level.

Q = the discharge.

R = a staff reading.

t = time, in seconds, counted from the instant of high water at the canal entrance.

t_m = time at which maximum high water occurs at the point, x. It is, therefore, the time required for the resultant wave to be propagated from the entrance of the canal to the point, x. t_1, t_2, t_3, etc., denote the time required for the wave of translation and the second, third, etc., reflected waves to reach the point, x, counted from the instant, $t = 0$.

t_{mv} = time of occurrence of the maximum velocity at a point, x. It is, therefore, the time required for the maximum resultant velocity to be propagated from the entrance of the canal to the point, x.

T = total tidal period from one high water to the next (44 715 sec.).

t_x = time required for the crest of the wave to travel from the entrance of the canal to the point, x, in question.

t_y = time required for the propagation of a reflected negative wave from the reservoir toward the sea to a point, x.

t_L = time required for the positive wave to be transmitted from the sea to the reservoir.

u = velocity of any current of water not of tidal origin.

U = horizontal displacement of the water molecules.

v = actual velocity of the water at any time, t, and at any point, x.

V_m = maximum velocity at any point corresponding to the time, t_{mv}.

V_0 = velocity at the entrance to the canal corresponding to the height, H_0. V_1, V_2, V_3, etc., equal the velocities at any point, x, due to the passage of a wave of translation, or the second, third, etc., reflected simple waves of translation, with heights equal to H_1, H_2, H_3, etc.

V_h = velocity due to height, h, in an infinitely long canal.

x = abscissa on the sine curve, or the distance required for wave propagation in the canal.

y = ordinate on the sine curve.

Y = height of water above the mean level of the canal, due to the propagation of a wave of translation at any instant, t, counted from the time of high water in the tidal sea.

ρ = density of the water.

Waves

To treat of this subject, it is first necessary to determine the character of the flow through canals subject to tidal influences. Most textbooks treat only of the flow of water under the influence of a surface slope, without much reference to tidal influences and the manner in which their effects are manifested. Tidal currents are sometimes attributed to the effects of the surface slope of the tidal wave, but both theory and observation disprove that explanation. Omitting from consideration for the present all questions of fresh-water flow or discharge, and taking into consideration only the flow directly attributable to tidal influences, consider the manner in which such influences are manifested.

All tidal action in coastal waters is clearly generated by the great ocean tidal wave formed directly under the combined attraction of the moon and the sun. This great wave gives rise to other waves which are propagated in greater or less degree to all waters connected with the sea at a suitable level. This paper is not concerned with any of the theories of the action of the ocean tide, except as it is manifested along the coast line; usually in a semi-diurnal rise and fall, during which the graph of the surface variations is approximately a cosine curve.

The only waves of importance affecting tidal canals besides the tidal waves are wind waves, or secondary waves generated by wind waves, and these are too irregular and variable to be considered herein.

Waves of Oscillation.—Every force which acts upon a liquid mass develops waves in it, among which waves of oscillation and waves of translation may be distinguished.

Waves of oscillation occur whenever the water is subjected to a vertical force which causes a momentary depression or elevation upon a relatively extensive area of the surface. They may be ordinary or periodical. Periodical waves follow each other like ordinary waves, but each has its own particular cause; their periodicity is inherent in the force which acts upon the water, and which produces a wave each time it acts. The undulations of the water oscillate above and below the level of equilibrium, and are of the same height or semi-amplitude in both directions.

The agitation produced by waves of oscillation makes itself manifest below the surface of the water. Undulations of the same length as those upon the

surface occur in the body of the water, but their amplitude diminishes from the surface. Waves of oscillation, therefore, cause motion in the water only for a limited depth.

Two different undulatory motions, which occur together in a liquid mass, are propagated independently of each other, without coalescing. The resultant wave has a different form from that of the two components; the variation of the level of the water at any given point is the algebraic sum of the superelevations and depressions due to the two waves. The heights are added together or subtracted according as they are above or below the mean level.

Waves of Translation.—A wave of translation will originate in the waters of a canal, at the extremity of which there has been introduced suddenly a given mass of liquid, or when there has been impressed upon a transverse section an instantaneous effect of compression. Thus, a laden boat in motion, on entering into a narrow section of the canal, of sufficient length, carries before it a certain quantity of water, and should the boat stop abruptly, the intumescence thus formed above the surface of the water detaches itself, becomes an undulation and continues its progress in the direction of the motion of the boat.

The wave of translation is single for each effort imposed upon the liquid mass; for this reason it is also called the solitary wave. It has the characteristic of being contained entirely above the surface of the water.

The most notable experiments which have been made upon the solitary wave date from 1837 and are due to Sir Scott Russell;* the results which he obtained have been confirmed by Bazin.† The works of these two scholars have disclosed the different properties of waves of translation.

Although the horizontal force which creates such a wave may act upon only a portion of the water, the wave spreads over the entire width of the surface, and puts the entire mass of liquid in motion from top to bottom.

When the wave is propagated in a canal of variable cross-section, it takes a greater height at the narrow points, while it becomes lower in the places where the canal is wider; but it always occupies the entire section of the canal.

In still water of sufficient depth each wave produced by a force exercising pressure on the liquid mass in the horizontal direction gradually takes a certain form which is distinguished by its great stability; this is influenced only by the attrition resulting from friction, which is quite slow.

The speed or velocity of propagation of the wave of translation, observed under these conditions, has for its expression,

$$C = \sqrt{g\,(D + h)} \qquad (1)$$

If the depth is variable in the same cross-section, the wave takes a single velocity of propagation corresponding to the mean depth of the section. In a wide rectangular canal, the waters of which are endowed with uniform motion, any wave of translation, when propagated in the direction of the current, takes a regular form, of remarkable longevity, which tends to approach that of the

* Rept., Fourteenth Meeting, British Assoc. for the Advancement of Science, September, 1844, Lond., 1845.

† "Recherches Expérimentales sur la Propagation des Ondes," *Mémoires* des Savants Etrangers, Tome XIX.

solitary wave traversing still water of constant depth. Its speed is given by the formula,

$$C = \sqrt{g\ (D + h)} + u \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For a wave which is propagated in a direction opposed to that of the current the formula becomes,

$$C = \sqrt{g\ (D + h)} - u \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In this case the wave soon departs from the original regular form, differing more as the current is swifter, and when the speed has diminished to the point of approaching the velocity of the current, it loses its vitality rapidly, the mass of water composing it being gradually dissipated in the current.

Boussinesq has determined by analysis a general theoretical formula which permits the computation of the speed of each layer, $d\,x$, of a wave of translation, when the wave has not acquired the permanent form which is assumed in the case of the experimental formulas, Equations (1), (2), and (3), which assume the same speed for all layers. Boussinesq's formula* is,

$$C^2 = g\left(D + \frac{3\,h}{2} + \frac{1}{h}\,\frac{D^3}{3}\,\frac{d^2\,h}{d\,x^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Among all the intumescences of various forms the propagation of which may be imagined, there is one for which the heights, h, and the curvatures, $\frac{d^2\,h}{d\,x^2}$, are so distributed that the speed of propagation is the same for all its elements. That is the solitary wave—the wave of translation—studied by Sir Scott Russell.

Tidal Waves in the Sea.—Tidal waves in the open sea take the form of waves of oscillation. The propagation of the wave sets up currents due to the displacements of the liquid involved in the motion. Tidal currents have no relation to the surface slope of the water; they are exclusively undulatory phenomena.

Tidal currents in shallow waters near the shore line are mass currents, acting for the entire depth of the sea; they are directed successively in the direction of travel of the undulation, and then in the opposite direction. They are called flood currents in the former case and ebb currents in the latter. They are alternating currents; barely perceptible in the deep sea, they increase in intensity as the depth of water diminishes. The currents sometimes become gyratory because of the combination of longitudinal currents with lateral currents which come and go, corresponding to the alternate rise and fall of the water on shore, or of the crossing of two co-existent waves the directions of which intersect at an angle.

The stage of the tide that corresponds to the moment when flood and ebb currents cease to run (and when they are about to be replaced by currents in the opposite direction) has received the name "flood slack", or "ebb slack", as the case may be. Under normal conditions the tidal wave is periodical and its curve takes a regular sinusoidal form; the currents which it generates turn

* "Essai sur la Théorie des Eaux Courantes," par J. Boussinesq; see, also, "Hydraulique," par A. Flamant, Paris, 1923, pp. 433–435 and pp. 438-440.

at mean tide level, reaching their maximum velocity or strength at high or low water. These phenomena are completely modified when the depth decreases rapidly, and the propagation of the wave is hindered or checked in its travel, as in approaching the coast line, in estuaries, and especially in tidal rivers.

In general, near the coast line, the situation is intermediate between the state of free propagation of the tidal wave, and that of filling a basin of small dimensions; the regimen of the tide becomes changed. The reversal or turn of the tide is then no longer effected at mid-tide, but approaches closer to the times of high and low water, and the maximum velocity approaches mid-tide. The velocities then differ from their theoretical value and, in order to interpret the phenomena observed, one is tempted at times to have recourse to considerations based upon the hydraulics of running water and the influence of surface slope. Under these conditions the tidal wave loses its usual characteristics, entirely or in part, and in order to account for observed facts, it must be considered as formed by a series of small waves of translation each of which is propagated according to its own proper law.

Waves in Tidal Channels.—In the propagation of a wave there is only a simple transmission of motion, during which the molecules of water are successively raised and lowered on the passage of a wave, without participating in any way in its velocity of propagation. These transmissions of motion cannot take place without being accompanied by transportation of liquid molecules occurring step by step in the horizontal direction, which gives rise to currents in the direction of propagation.

For waves in tidal channels, the horizontal force which produces them resides in the rise of the tide at the mouth. The derived wave in tidal channels is not a tidal wave in the sense in which that term is used as to the ocean tidal wave; it is not a periodic wave of oscillation, a chop, or a seiche. It exhibits the traits of a simple wave of translation, resembling it notably in the means of its formation, but differing from it as soon as the conditions of propagation are considered. The velocity of propagation of the derived wave is usually less than is indicated by the usual theoretical formulas. Furthermore, in the true simple wave of translation, all the velocities are in the same direction, whatever point of the wave is considered, while in the derived wave the currents flow alternately in one direction and then in the other, and the velocities may be greater or less than for a simple wave of translation.

The various discrepancies, however, are more apparent than real, and arise from the fact that formulas for the speed and velocity of the wave of translation, such as Equations (1) to (4), inclusive, are applicable to small waves which are propagated in relatively wide and deep channels. In cases of this kind the influences of resistances and of reflections are inappreciable. The equations are not applicable to waves for which the height is comparable to the depth of the channel and which are propagated in short, crooked channels with irregular banks. These conditions offer very great resistance to the propagation of the wave. It is to these resistances and to reflections within the channel that the discrepancies observed between the behavior of the derived wave and that of small simple waves of translation, must be attributed.

Effect of the Canal on Waves in It.—Accepting these conclusions as to the identity of the canal wave with the wave of translation, such a wave entering into a canal from another body of water will be confronted by a set of conditions differing from those prevailing when it enters a tidal river or estuary from the sea. Its character is changed by the expenditure of energy necessary to overcome each resistance to its progress, and this expenditure must then cause either a diminution of the volume of the wave, a reduction of its height, or both simultaneously.

A tidal river or estuary will always be found closed at its upper end. This limits the distance in which the wave may be propagated and it limits the volume of the wave that may be propagated past any given point. In such cases the wave may also be opposed by a large fresh-water discharge. In addition to friction losses, the usual funnel shape of the river or estuary causes a continual decrease in the volume of the wave. Such lost volume, together with friction losses and any fresh-water discharge, is ejected down stream.

These conditions, however, are not usually encountered in a canal. The canal is not ordinarily of a funnel shape; it is usually of constant depth and cross-section. There is seldom any fresh-water discharge to be considered.

The physical characteristics of a canal have a preponderating influence upon the propagation of the tide within it. Assuming a horizontal canal of constant width and depth, communicating at one end with a body of tidal water, the character of the tide within the canal will be further controlled by the length of the canal and the character of the body of water with which it communicates. In this study it will be assumed that the canal communicates at its far end with some body of water, and is not merely a closed channel.

Regardless of the length of the canal, three cases will arise with respect to the body of water with which it communicates:

(1) The body of water may be at a constant level;
(2) It may vary in level due to tidal flow through the canal; and
(3) It may vary in level due to a proper tide of its own.

Case (1), as stated, implies a non-tidal reservoir sufficiently large in area so as not to be affected appreciably by any tidal flow it may receive through the canal. Case (2) may be a rather small reservoir which will be partly filled and emptied through the canal at each tide. In Case (3) the canal will be the meeting place of two opposed independent waves, the phases and amplitudes of which may differ widely.

These three comprise practically all cases of tidal flow in canals. The flow in tidal rivers and estuaries will not be included in this study. However, before beginning these practical studies, it is necessary to consider the theory of tidal action in a canal of infinite length, in order that the theory may be of assistance.

The Theoretical Canal, Infinitely Long

An infinitely long canal is a fiction that has been invented to meet the necessities of theory. Except in rare instances, such as the Amazon River, the St. Lawrence River, or Chesapeake Bay, the length of channel influenced by the tide does not exceed that of an entire wave. It is nevertheless of interest

to follow scientists into the hypothetical domain and to examine the theoretical regimen of the tide in such a canal assuming that its bottom is horizontal and that its cross-section remains constant for its entire length. Under these restrictions the question has been treated by several investigators, and it is of interest to review the results at which they have arrived.

Theory of Airy.—Airy has devoted an important work* to the general study of the tides and of waves of different kinds, in which he has discussed in the greatest detail the circumstances of the propagation of the tidal wave in tidal rivers and estuaries. The results which he obtained may be summed up in the following equations:

$$h = H_0 \sin \frac{2\,\pi}{T}\left(1 - \frac{x}{\sqrt{g\,H_0}}\right) + \frac{3}{4}\,\frac{2\,\pi\,H_0^{\,2}}{T\,D} \sin \frac{4\,\pi}{T}\left(t - \frac{x}{\sqrt{g\,H_0}}\right) \ldots (5)$$

The speed or velocity of propagation of the wave equals, at high-water stand,

$$C = \sqrt{g\,(D + 3\,H_0)} \ldots\ldots (6)$$

and at low-water stand,

$$C = \sqrt{g\,(D - 3\,H_0)} \ldots\ldots (7)$$

The horizontal displacements of the molecules, which are the same from the bottom to the surface, are:

At high-water stand,

$$v = \left(1 - \frac{5\,H_0}{8\,D}\right)\frac{H_0}{D}\sqrt{g\,D} \ldots\ldots (8)$$

and at low-water stand,

$$v = \left(1 + \frac{5\,H_0}{8\,D}\right)\frac{H_0}{D}\sqrt{g\,D} \ldots\ldots (9)$$

Airy gave the changes to be made in these formulas when the water has a seaward current and when its depth is variable. He recognized that a bore should occur in such a case unless an additional horizontal force should intervene to combine its action with that of gravity, and he suggested that friction might supply such a force. He attempted to evaluate the influence of friction of the liquid on the bottom of the canal, assuming it to be proportional to the velocity, but he found that an evaluation thus made, did not possess the desired reliability for use in practice; nor was it capable of explaining the phenomena noted by observation.

Theory of Saint-Venant.—Saint-Venant has treated the same question† based upon a hypothesis previously proposed by Bremontier and confirmed by Bazin and Partiot. It consists in considering the tidal wave (which is of the class termed "periodical") as the result of the superposition of a series of very small waves of translation being propagated according to the laws established by Scott Russell and developed by Bazin. During the flood wave, these elementary waves would be formed by intumescences or fluxes, and during the ebb by depressions or refluxes.

* "Tides and Waves," Encylopedia Metropolitana, Vol. V, 1845, p. 241.
† *Comptes Rendus* de l'Academie des Sciences, Vol. LXXIII, 1871, pp. 147 and 189.

Under this hypothesis, and neglecting the influence of friction upon the walls of the canal (assumed to be rectangular), he obtained the following results:

$$v = 2\sqrt{g\,(D + h)} - 2\sqrt{g\,(D - h)} \ldots\ldots\ldots\ldots\ldots (10)$$

$$C = 3\sqrt{g\,(D + h)} - 2\sqrt{g\,(D - h)} = v + g\sqrt{(D + h)} \ldots\ldots (11)$$

and,

$$x = C\,(t - \frac{T}{\pi}\ \text{arc sin}\sqrt{\frac{2\,h}{D}} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Theory of Maurice Lévy.—The theory of Saint-Venant was verified and confirmed by Maurice Lévy* who also treated the general question of the regimen of the tide in canals both of constant and of variable width. He made an application of the theory to river tides in the case in which the width is constant or slowly variable, due regard being given to the friction exerted upon the tidal wave.

He established the principles that:

1.—The wave is propagated with a decreasing height according to a definite coefficient of extinction.

2.—The velocity of propagation is given by a simple expression as a function of the height of the tide and of the depth of the water in the canal at half-tide.

3.—The strength of the current occurs before the high and the low waters, and its value diminishes as the point considered is farther from the mouth.

His results are better in accord with the results of observation than those of the others. The superiority of his theory arises principally from the fact that Lévy has succeeded in taking into account the effects of friction which Saint-Venant had neglected and which Airy thought inadequate to explain the discrepancies between facts and the results of his analysis. This influence of friction upon the phenomena of the tide, formerly regarded rather vaguely, is now an established fact. It can be computed in a fairly precise manner, under simple circumstances, it is true, but nevertheless in such a way as to permit rational inductions to be made for the most complicated cases.

In attempting to apply his equations to the Suez Canal, Lévy found that the computed values of the velocity were far less than the actual velocities, particularly at the end next to the Bitter Lakes. He attempted to remedy this discrepancy by the use of equations to a second approximation, but still without a complete agreement with observation.

The theory of Lévy is quite fully set forth and explained in an article on the Cape Cod Canal by William Barclay Parsons,† M. Am. Soc. C. E. This paper gives the deduction of Lévy's equations and makes a practical application of them to the Cape Cod Canal. Recognizing that the frictional coefficient is not a constant quantity, but varies slightly in an inverse ratio with the velocity, Mr. Parsons proposes a frictional coefficient which, as a linear func-

* "Leçons sur la Théorie des Marées," par Maurice Lévy, Gauthier-Villars, 1898.

† *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), pp. 1–157.

tion of the mean velocity, will absorb the same total energy in the interval during which the velocity changes from zero to maximum.

Writings of Bourdelles.—In a remarkable series of memoirs,* M. Bourdelles, Inspector-Général des Ponts et Chaussées, has fully and clearly set forth the theory of the propagation of waves of translation in canals, and has pointed out how closely observed facts confirm that theory. Bourdelles' exposé of the theory is so clear and vigorous that much of what follows has been adapted from his writings. However, Bourdelles, as well as Lévy, was unable to explain satisfactorily certain anomalies of velocities observed at points where canals empty into tideless seas. In such cases, where the observed velocities exceeded the computed velocities, those authors explained the anomaly by attributing it to the influence of flow due to surface slope in the undulatory motion of the tide. The writer advances the theory of a reflected negative wave to explain it.

Tidal Rivers.—M. Bonnet, Chief Engineer and Director of the Ponts et Chaussées of Belgium, has made a most valuable contribution† to the theoretical study of tidal rivers. He treats the propagation of the tidal wave as a wave of translation continually modified by friction (which he evaluates) and by the shape of the bed. These causes which act to reduce the energy of the wave give rise to a current directed down stream the volume of discharge of which represents at each instant the volume of water lost in a unit of time by the portion of the wave which has passed into the river. This he terms counter-current. He also shows that the propagation of the wave of translation and the action of the counter-current permit an explanation of all the characteristics of the river tide. Bonnet's method of showing how the energy of the wave is reduced by friction has been adopted in this paper, with necessary modifications.

Propagation of Elementary Waves of Translation in an Infinitely Long Canal.—Assuming that the tidal wave entering a canal is a wave of translation, it is necessary to consider some of the properties of the latter. To do so intelligently, the first step is to study the propagation of such a wave in a canal of constant depth and cross-section, but of infinite length.

Suppose the water is without motion at the moment of the introduction of the first flux emitted by the tidal wave on the flood. In Fig. 1, which represents a longitudinal profile of the canal, $A\,B$ is the mouth with a mean depth, D; $A\,M$, the horizontal level of the water in the canal at low water; and $h = A\,a$, the height of the first flux the propagation of which is being studied.

By virtue of the properties of the wave of translation, after a time which will be assumed as unity, this flux, propagating itself with a speed of $C = A\,M$, will have arrived at $M\,N$, introducing into the canal a volume, $A\,a\,m\,N = C \times h$ per unit of width, and in impressing upon all the vertical filaments of the canal a horizontal displacement of translation the velocity of which will be designated by v. This displacement will have the effect of moving the mole-

* Bourdelles contributed the following articles published in *Annales des Ponts et Chaussees*: (1) "Etude du Regime de la Maree dans le Canal de Suez," 1898, 3[me] trimestre; (2) "Distribution des Vitesses Suivants la Verticale dans les Courants de Maree," 1898, 4[me] trimestre; (3) "Etude du Regime de la Maree dans la Manche," 1899, 3[me] trimestre; and (4) "Etude sur le Regime de la Maree dans les Estuaires et dans les Fleuves," 1900, 2[me] trimestre.

† "Contribution à l'Etude Théorique des Fleuves à Marée," par L. Bonnet, *Annales des Travaux Publiques de Belgique*, 1922, Fascicules 3, 4, 5, and 6, and 1923, Fascicules 1, 2, 3, and 5.

cules of water of the section of the mouth, $B\,A$, to $B'\,A'$, and of introducing into the canal a volume, $B\,a\,a'\,B' = v\,(D + h)$. Since this volume is necessarily equal to that of the flux, $C\,h = v\,(D + h)$, or,

$$v = C\,\frac{h}{D + h} \qquad (13)$$

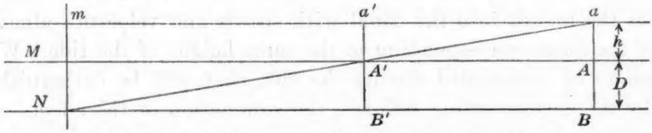


FIG. 1.—LONGITUDINAL PROFILE OF A CANAL.

Thus, a known relation is found between the velocity of displacement common to all the molecules and the speed of the undulatory motion which causes this displacement. Furthermore (see Fig. 1), the velocity is represented by the right line. $A\,A'$, since $\dfrac{h}{D + h} = \dfrac{A\,A'}{B\,N} = \dfrac{A'\,A}{C}$. The effect of the undulatory motion thus becomes quite clear. The impetus is propagated successively to all the vertical filaments of the canal with a rapidity equal to the speed, giving to each of these filaments the same velocity, so that the compression of the water comprised between two neighboring filaments produces the height of the flux corresponding to it. Hence, the total compression, equal to the volume of the flux, provokes an equivalent supply of water through the mouth.

Fig. 1 also demonstrates that, for the same depth, D, the supply of water to the canal and the horizontal displacements increase with the height of the flux. The same is true when (the height of the flux and the mean depth remaining constant) the speed is nevertheless increased by extraneous circumstances.

It is to be noted finally that, in the infinitely long canal, the first flux will continue to be propagated during the entire period of the flood until the moment when the ebb, beginning to be formed at the mouth, will be opposed to the introduction of the necessary water for the supply of the flux. In spite of this obstacle, it will be able, however, to continue its propagation to some degree; but then the action of friction will result in the gradual diminution of its height and its final extinguishment.

It is needless to add that all that precedes is also applicable to the refluxes on the ebb. There is no difference except in the lesser speed of the propagation and also in the mode of supplying the wave volume. This is effected by expelling water from the canal, creating a current which is opposite in direction to that of propagation.

Friction has the effect of diminishing the height, the speed, and the velocity of a flux or of a reflux, by an amount proportional to the velocity and to the duration of the undulatory motion.

The first flux will be propagated during the entire flood tide, under the conditions just indicated. The second flux will be propagated in the same manner as the first, with the difference, however, that it will flow in the

current created by the preceding flux, which will increase its normal speed by a quantity equal to the velocity of that current. It is the same for the other fluxes which follow, up to the moment of high water, so that the velocity of the flood current will continue to increase in all parts of the canal during the rising tide.

Beginning with the moment when the tide begins to fall, the refluxes will travel from the mouth into the canal with speeds and velocities almost equal to those of the fluxes corresponding to the same height of the tide. When the fluxes cease to be propagated during the ebb, they will be extinguished successively by the corresponding refluxes, the summation of the effects of which will accordingly be to evacuate to the sea, almost completely, a quantity equivalent to the water introduced by the flood. The fluxes continue their propagation, however, when the length of the canal allows it, and they will escape entirely from the action of the refluxes if the length of the canal is infinite. Under the hypothesis as defined, therefore, it is apparent how a portion, or even all, of the volume, introduced by the fluxes, may remain in the canal after the end of the ebb, without being evacuated by the succeeding refluxes. Then the surface of the canal becomes a succession of positive and negative waves of translation, both being propagated in the same direction. The positive waves leave each particle of water moved forward a distance dependent upon the volume of the wave, and each succeeding negative wave, if it is of equal volume, returns the particle to its original location. If for any reason, such as increased frictional resistances, the negative wave should not equal the positive wave in volume, the particle will suffer a permanent displacement in the direction of propagation after the passage of each positive wave and its succeeding negative wave.

Friction.—The sum of all friction losses in the water flowing in an open canal—whether flowing under the influence of a surface slope or under the impulse of the propagation of a wave of translation—may be reduced to the friction on the sides and the bottom of the canal, which is to say, on the wetted perimeter.

Observation has resulted in the formulation of general laws as to friction losses. For example, they are proportional to the length of the channel; they increase with the roughness of the walls; they decrease as the mean depth increases; they increase nearly as the square of the velocity; and they are independent of the pressure of the water.

It is usually assumed that the friction per unit of wetted surface may be represented by,

$$F = f_0 v^2 \qquad (14)$$

In this study, F will be considered as a constant for any canal, and will be given such value as shall have been found experimentally sufficient to reduce the summit of each wave to its observed value.

The reason for this is that the coefficient of friction is applied only to the value of the maximum height or summit of the wave to determine its value at a given point in the canal, and the remaining heights are assumed to be found from this maximum height by the cosine curve. As this maximum height

varies quite slowly from point to point under the influence of friction, the corresponding maximum velocities do not vary rapidly. Accordingly, the value of the frictional coefficient varies slowly enough to be considered as constant without any great error.

The influence of friction is manifestly of more importance as the depth of the channel becomes smaller. In a narrow strait, several hundred feet deep, it might be negligible, but in artificial canals and the more prevalent tidal channels, it has a great effect.

Effect of Friction in Reducing Height of Wave in an Infinitely Long Canal.—It has been established experimentally that, in the case of a long flat wave of translation, with little curvature (as in the case of a tidal wave of translation), the speed of propagation, C, may be found from the so-called Scott-Russell formula (Equation 1). The current velocity, v, may be represented by,

$$v = C \frac{H}{D + H} \qquad (15)$$

When the velocity of propagation is not very great, and when only the interior frictional resistances of liquids are considered as a main cause, the resistance due to friction per unit length of wave is equal to $F\,p\,v$.

The theoretical value of v is expressed by,

$$v = \sqrt{g\,(D + H)} \times \frac{H}{D + H} = \frac{\sqrt{g}\,H}{\sqrt{D + H}} \qquad (16)$$

Hence,

$$F\,p\,v = \frac{F\,p\,\sqrt{g}\,H}{\sqrt{D + H}} \qquad (17)$$

The elementary work of frictional resistances during the time, dt, represented by dF', is,

$$dF' = F\,p\,v \times v \times dt = F\,p\,v^2\,dt = \frac{F\,p\,g\,H^2\,dt}{D + H} \qquad (18)$$

The work, dF', represents the loss of energy of the tidal wave per unit of length in the time, dt. Hence,

$$dE = -\,dF' = -\,\frac{F\,p\,g\,H^2\,dt}{D + H} \qquad (19)$$

It can be shown from theoretical considerations that the energy of any section of wave of unit length may be expressed as,

$$E = \rho\,g\,l\,H^2 \qquad (20)$$

If x is the distance of propagation in the channel, $\frac{dx}{dt} = C = \sqrt{g\,(D + H)}$; whence,

$$dt = \frac{dx}{\sqrt{g\,(D + H)}} \qquad (21)$$

Differentiating Equation (20), substituting for dt in Equation (19) its value from Equation (21), and simplifying,

$$dE = \frac{(D + H)^{\frac{3}{2}}\,dH}{H} = -\,\frac{F\,p\,dx}{2\,\rho\,l\,\sqrt{g}} \qquad (22)$$

By integrating Equation (22), an expression is obtained which shows how the height of a wave of translation in a canal of infinite length and constant section is reduced from the effects of frictional resistance as the wave is propagated along the axis of x.

Equation (22) may be expanded and reduced as follows:

$$\frac{D^{\frac{3}{2}}\, dH}{H} + \frac{3}{2} D^{\frac{1}{2}}\, dH + \frac{3}{8}\frac{H\, dH}{D^{\frac{1}{2}}} - \frac{1}{16}\frac{H^2\, dH}{D^{\frac{3}{2}}} + \ldots = -\frac{F\, p\, dx}{2\, \rho\, l \sqrt{g}} \quad (23)$$

As the expanded equation is rapidly convergent, all terms after the third may be omitted in practice. Integrating Equation (23),

$$D^{\frac{3}{2}} \log_e H + \frac{3}{2} D^{\frac{1}{2}} H + \frac{3}{16}\frac{H^2}{D^{\frac{1}{2}}} - \frac{1}{48}\frac{H^3}{D^{\frac{3}{2}}} + \ldots = -\frac{F\, p\, x}{2\, \rho\, l \sqrt{g}} + \text{a constant} \quad (24)$$

When $x = 0$, let $H = H_0$; then,

$$D^{\frac{3}{2}} \log_e H_0 + \frac{3}{2} D^{\frac{1}{2}} H_0 + \frac{3}{16}\frac{H_0^{\,2}}{D^{\frac{1}{2}}} - \frac{1}{48}\frac{H_0^{\,3}}{D^{\frac{3}{2}}} + \ldots = \text{a constant} \quad (25)$$

Subtracting Equation (24) from Equation (25),

$$D^{\frac{3}{2}} \log_e \left(\frac{H_0}{H}\right) + \frac{3}{2} D^{\frac{1}{2}} (H_0 - H) + \frac{3}{16}\frac{(H_0^{\,2} - H^2)}{D^{\frac{1}{2}}} - \frac{1}{48}\frac{(H_0^{\,3} - H^3)}{D^{\frac{3}{2}}} + \ldots = \frac{F\, p\, x}{2\, \rho\, l \sqrt{g}} \quad (26)$$

which is the desired formula.

Equation (26) shows that the wave would be propagated in an infinitely long canal for an infinite distance; that is, theoretically, the continually reducing effect of friction would cause the wave to disappear only at an infinite distance; but, practically, it would disappear within appreciably finite distances. Except for friction the wave would be propagated indefinitely at full volume. However, the effect of friction is continually to reduce the volume of the wave by a varying quantity per foot. Assuming for the moment that this lost water could be held at the point at which it was left behind by a single positive wave, it would appear upon the original canal surface as a thin wedge, thickest at the canal entrance, and diminishing to zero only at an infinite distance. Actually, of course, this water left behind by the wave in its travel will tend to flow back to the source and add its volume to that of the succeeding negative wave.

Since waves of translation may be either positive or negative, a depression in the water surface may be propagated as a wave, as well as an intumescence, in which case the height of the wave, h, will have a negative value. Since Equation (26) is of general application, by giving proper positive and negative values to H, curves will be found which will be the theoretical loci of high and low tide for the infinitely long canal. (See Fig. 2.) It is apparent at once that these curves are not symmetrical. The effect of friction in destroying the wave is greater for the negative than for the positive wave, since the depth of water is less in the canal during the propagation of such a wave.

Time Required for Crest of Wave to Reach a Given Point.—If Equation (21) is written, $\frac{x}{t} = C = \sqrt{g\,(D + H)}$, the rate of propagation to a given point is found to depend on the height of the wave which, due to the action of friction, is variable. If, however, the values of C are computed for given values of H selected quite close together, the rate of propagation, C, may be assumed constant between these points, and a mean value for the points in question may be assumed as correct without appreciable error. This gives a new value, C', for the rate of propagation between two such points,

$$C' = \frac{1}{2}\sqrt{g}\left[\left(\sqrt{D + H_1}\right) + \sqrt{D + H_2}\right] = \frac{(x_1 - x_2)}{t} \quad \ldots\ldots(27)$$

whence,

$$t = \frac{2\,(x_1 - x_2)}{\left(\sqrt{D + H_1} + \sqrt{D + H_2}\right)\sqrt{g}} \quad \ldots\ldots\ldots\ldots\ldots(28)$$

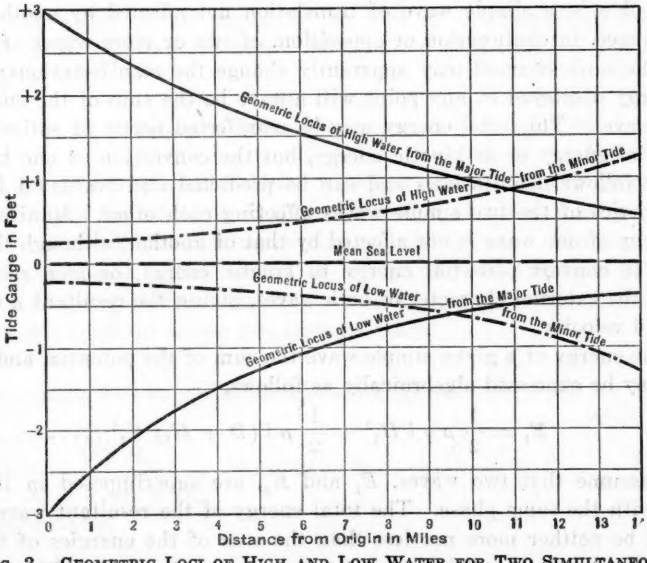


FIG. 2.—GEOMETRIC LOCI OF HIGH AND LOW WATER FOR TWO SIMULTANEOUS WAVES PROPAGATED IN A CANAL OF INFINITE LENGTH.

For the negative wave, of course the values of H in Equation (28) would be negative.

The equations show that, considering friction, the rate of propagation, or the speed, falls off as the wave progresses, and the wave constantly tends to check its speed. Furthermore, this effect is greater for the negative than for the positive wave.

If, therefore, a series of successive waves of translation, positive and negative alternating, are being propagated in the canal one would expect to find the positive waves tending to outrun the negative waves slightly, the effect being the more apparent the greater the distance traveled.

Energy.—The total energy of a wave of translation,* or the work which it could do in being brought to a state of rest, would be constant were it not for friction; but while continually reduced by friction, the remaining energy at any point of an indefinite canal consists of two equal parts, potential energy as represented by the height of the intumescence above the mean level, and kinetic energy as represented by the velocity of the mass of water. The expression for the potential energy of one unit of length of wave is $\frac{1}{2}\ \rho\ g\ l\ h^2$, and that for its kinetic energy is $\frac{1}{2}\ \rho\ l\ (D\ +\ h)\ v^2$. The equality of these two kinds of energy may be made apparent by substituting in the latter expression for v its value from Equations (13) and (15). This equality of energy may be upset, however, by any cause which impresses a change upon either height or velocity of the wave.

The equality of these two manifestations of energy is absolute and unchangeable in a simple wave of translation not affected by another wave. The existence, in conjunction or opposition, of two or more waves of translation in the same channel may apparently change the manifestations; but the total energy displayed at any point will always be the sum of the energies of the two waves. This total energy may be manifested partly or entirely either as potential energy or as kinetic energy, but the conversion of one kind into the other follows definite laws and can be predicted and evaluated from the characteristics of the two simple waves affecting each other. Manifestly, the total energy of one wave is not affected by that of another, although the latter may act to convert potential energy to kinetic energy, or *vice versa,* thus changing the external character of both waves, giving the resultant a different height and velocity.

For the energy of a given simple wave the sum of the potential and kinetic energy may be expressed algebraically as follows,

$$E_1 = \frac{1}{2}\ \rho\ g\ l\ H_1^2 + \frac{1}{2}\ \rho\ l\ (D + H_1)\ V_1^2 \text{.............} (29)$$

Now assume that two waves, E_1 and E_2, are superimposed in the same channel with the same phase. The total energy of the resultant wave at this point can be neither more nor less than the sum of the energies of the component waves, neglecting friction; hence,

$$E_3 = E_1 + E_2 = \frac{1}{2}\ \rho\, g\, l\, (H_1^2 + H_2^2) + \frac{1}{2}\ \rho\, l\, (D + H_1 + H_2)\, (V_1^2 + V_2^2) \text{..} (30)$$

It will be noted that in Equation (30) the total resulting depth of water $(D + H_1 + H_2)$, is used instead of $(D + H_1)$ or $(D + H_2)$. In compound waves, the total depth must always be used in computing the respective velocities of component waves from their respective heights. Failure to do so would involve an error in the expression for the total amount of kinetic energy.

It will be interesting to see whether any change as between kinetic and potential energy has occurred in this superposition. Assume, in accordance

* "Hydraulique," par Flamant, Paris, 1923, p. 434.

with the usual understanding as to the formation of a resultant wave from two components, that $E_3 = E_1 + E_2$; $H_1 + H_2 = H_3$; and $V_1 + V_2 = V_3$. If the resultant wave is assumed to be a simple wave of translation, its equation will be similar to Equation (29), thus:

$$E_3 = \frac{1}{2}\,\rho\, l\, g\, H_3^2 + \frac{1}{2}\,\rho\, l\,(D + H_3)\,V_3^2 \ldots\ldots\ldots\ldots\ldots (31)$$

or,

$$E_3 = \frac{1}{2}\,\rho\, l\, g\,(H_1^2 + 2\,H_1\,H_2 + H_3^2) + \frac{1}{2}\,\rho\, l\,(D + H_1 + H_2)$$
$$(V_1^2 + 2\,V_1\,V_2 + V_2^2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Subtracting Equation (31) from Equation (30) to solve for V_3 and H_3, $V_3^2 = V_1^2 + V_2^2$ and $H_3^2 = H_1^2 + H_2^2$.

This result seems to be at variance with the understanding previously expressed, that when one wave is superposed upon another the resulting heights and velocities are equal to the sum of the respective heights and velocities of the component waves. In Equation (32) if $H_1 = H_2$, and hence $V_1 = V_2$,

$$E_3 = 4\left[\left(\frac{1}{2}\,\rho\, l\, g\, H_1^2\right) + \frac{1}{2}\,\rho\, l\,(D + 2\,H_1)\,V_1^2\right] \ldots\ldots\ldots (33)$$

which is twice as great as the actual energy available in the two waves. Such a result violates the principle of the conservation of energy and is manifestly incorrect. It invalidates the assumption that in the case of superposed waves the resultant height is the sum of the component heights, and the resultant velocity is the sum of the component velocities. In other words, the resultant wave can not conform to the assumptions made.

Next, assume the two equations expressing the relations, $V_1 = \dfrac{\sqrt{g}\,H_1}{\sqrt{D + H_1 + H_2}}$ and $V_2 = \dfrac{\sqrt{g}\,H_2}{\sqrt{D + H_1 + H_2}}$. Multiplying these together, $V_1\,V_2 = \dfrac{g\,H_1\,H_2}{D + H_1 + H_2}$, and, consequently,

$$\frac{1}{2}\,\rho\, l \times 2\,V_1\,V_2\,(D + H_1 + H_2) = \frac{1}{2}\,\rho\, l\, g \times 2\,H_1\,H_2 \ldots\ldots (34)$$

Subtracting Equation (34) from Equation (30),

$$E = \frac{1}{2}\,\rho\, l\, g\,(H_1 + H_2)^2 + \frac{1}{2}\,\rho\, l\,(D + H_1 + H_2)\,(V_1 - V_2)^2 \ldots (35)$$

It is apparent that if $H_1 + H_2 = H_3$, then must $V_1 - V_2 = V_3$, or if $V_1 + V_2 = V_3$, then must $H_1 - H_2 = H_3$, in order that the principle of conservation of energy may not be violated. From these results one may infer that any energy that may appear to have been added as a reinforcement to the actual height of the resultant wave must disappear as a loss from the total energy of the velocities of the component waves, or *vice versa.*

The general conclusions to be drawn from these facts are that:

1.—For two or more waves traveling together in conjunction, one wave is formed, the height and velocity of which are equal to the square root of the sum of the squares of the respective component heights and velocities.

2.—For waves traveling in opposite directions at the points of conjunction, if both waves are either positive or negative, the heights will be the sum of the heights of the component waves, and the velocity will be the difference between the component velocities.

3.—If one wave is positive and one negative—that is, if the waves are in opposition—the resultant height will be the difference between the component heights, and the resultant velocity will be the sum of the component velocities.

Influence of Change of Width.—Consider next the effect upon the tidal wave of an abrupt change in the width of a canal.

Let the canal, $A\,B$, in Fig. 3, with a width, l, suddenly become narrower at B, so that its width is l' in the section, $B\,C$. If h_0 is the height of the generating wave at A and h_1 is its height as reduced by friction at B, then the energy of the wave at B will be $E = \rho\, g\, l\, h_1^2$. Due to the reduced width of $B\,C$, only a portion of this energy can be propagated into $B\,C$. Let the actual height of the tide at B equal h; then this energy, $E_1 = \rho\, g\, l_1\, h^2$.

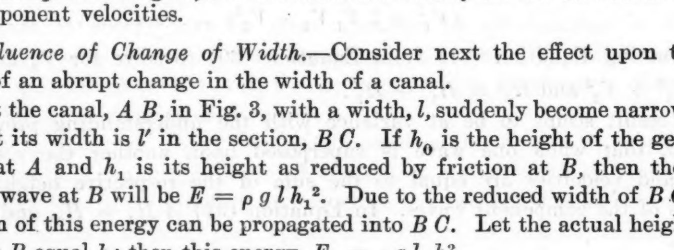


FIG. 3.—PLAN OF CANAL WITH SUDDENLY CONTRACTED SECTION.

The energy which is not transmitted past B will manifest itself in a reflected wave traveling back from B toward A, and will have the effect at B of raising the water level at that point higher than it would have been had the width been constant, and it accounts for the difference between h_1 and h.

Therefore, this reflected energy may be expressed by the formula,

$$E_2 = E - E_1 = \rho\, g\,(l\, h_1^2 - l_1\, h^2) = \rho\, g\, l\,(h - h_1)^2 \ldots\ldots\ldots (36)$$

Reducing,

$$l\, h_1^2 - l_1\, h^2 = l\, h^2 - 2\, l\, h\, h_1 + l\, h_1^2$$

and,

$$h = \frac{2\, l\, h_1}{l + l_1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

If l_1 is less than l, h will be greater than h_1; for example, if $l_1 = 0$, $h = 2\, h_1$. If l_1 is greater than l, h will be less than h_1; for example, if $l_1 = \infty$, $h = 0$. In the former case the reflected energy is positive and causes an additional rise in level between A and B. In the latter case, energy is abstracted from the canal, or the reflected energy is negative, and causes a lowering of level between A and B.

THEORY OF THE INTERFERENCE OF TWO OPPOSED WAVES

It is desirable to consider theoretically the results of superposing two tidal waves that are propagated in opposite directions in a horizontal canal of

uniform section and infinite length, neglecting the effects of friction.* It will be assumed that the waves are of the class called periodic, exactly sinusoidal, and of the same length, and that, conformably with theory, the velocities of the currents which they generate are approximately proportional in any vertical to the height of the wave referred to its mean level.

Under these conditions, it is easy to find the instantaneous profiles as well as the distribution of the velocities of the resulting wave during the entire period of its propagation. Considering as positive the heights of the two component waves above the mean level, it will only be necessary to add them algebraically to obtain the corresponding heights of the resulting wave. Likewise, the distribution of the velocities in the wave will be obtained by adding algebraically the velocities of the component waves in the same vertical. Those in the direction of the propagation of the wave which has the largest vertical amplitude, are considered positive and those which have an opposite direction are negative. There are then three relative positions of the two waves to be considered principally.

First consider what happens in such a case when the waves are in coincidence, or in conjunction.

In Fig. 4, let $A\,2\,C\,2\,E$ be a wave being propagated toward E, and let $E\,1\,C\,1\,A$ be a wave being propagated toward A. Since both waves are tidal, they will have equal lengths, and there will be points such as A, C, and E, at which their points of zero elevation will arrive simultaneously. It is evident that these points will be spaced at distances apart equal to one-half a wave length. Half way between these points, and, consequently, distant from them by one-quarter of a wave length, there will be points, B and D, at which the maximum height of each wave will arrive simultaneously.

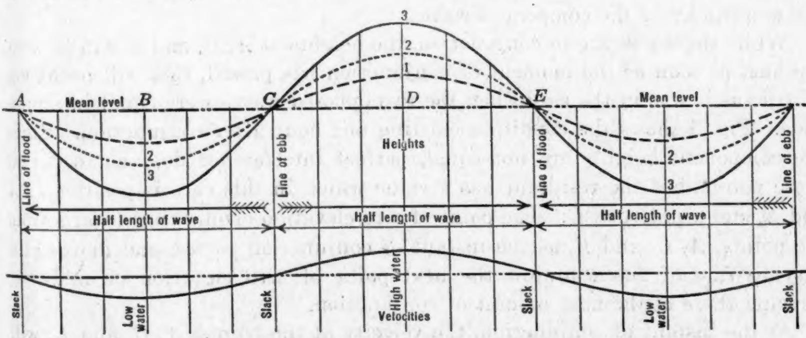


FIG. 4.—POSITION OF WAVES AT MOMENT OF THEIR CONJUNCTION.

Now, if the two opposed waves are exactly equal in amplitude, the sum of their heights at A, C, and E will always be equal to zero; that is, the waves will interfere perfectly at those points during the complete tidal period and there will be no variation in level, while at the half-way points the resultant wave will have an amplitude equal to the sum of their amplitudes, or to twice

* "Etude du Régime de la Marée dans La Manche," Bourdelles, *Annales des Ponts et Chaussees*, 1899, 3me trimestre.

their common amplitude. On the other hand, at *A, C,* and *E,* the respective velocities will add their values to give a new velocity equal to twice that of a single wave, while at *B* and *D* the velocities will annul each other, being equal in value and opposite in direction.

Then, the surface of the hypothetical canal, when subjected to the action of two equal and opposed waves, will present the appearance of being subdivided into sections the length of one half wave in which the surface oscillates up and down in amplitudes increasing from zero vâlue at *A* to a maximum at *B* and then decreasing to zero again at *C,* and so on indefinitely.

At the same time the mass of water in the canal will flow to and from the points, *B* and *D,* increasing in velocity in both directions from these points as a starting place to a maximum at *A, C,* and *E,* but always decreasing to zero again at *B* and *D* and reversing this action in the next half of the wave.

Points such as *A, C,* and *E* are nodes, and *B* and *D* are summits. Therefore, as a result of the interference of two opposed equal waves, there is a series of alternating nodes and summits, the surface variations being zero and the velocities greatest at the nodes, and the surface variations greatest and the velocities zero at the summits.

This state of affairs gives rise to a resultant wave termed the stationary wave or "chop." It is called stationary because the wave form does not progress in the canal, but remains fixed in position, and it is always characterized by these peculiarities. Its special laws will be summarized later.

For waves of unequal height in conjunction, the crests and the troughs of the two component waves will be found in the same vertical, the swells of the waves being in the same phase. (See Fig. 4.) In this position, the vertical amplitude of the resultant wave is a maximum and is equal to the sum of the amplitudes of the component waves.

While the waves are in conjunction, the heights at *A, C,* and *E* will be zero, but just as soon as the moment of conjunction has passed, this will no longer be true as it was in the case when the two opposed waves were equal in amplitude. Fig. 5 shows the condition existing one hour after conjunction. Since the component heights are not equal, perfect interference does not occur at those points, but the resultant has a value which in this case is positive at *A* and *E,* and negative at *C.* The point of zero elevation or mid-tide departs from the points, *A, C,* and *E,* as the instant of conjunction passes, and moves at a varying rate of speed toward the next point of zero elevation or mid-tide, arriving there at the next instant of conjunction.

At the instant of conjunction, the velocity at the points, *A, C,* and *E,* will be zero, or it will be slack water, while at the points, *B* and *D,* half-way between, where the tidal variation is greatest, there will not be a slack in the current as there would be if the two opposed waves were equal. At these points there will be, however, a velocity equal to the difference between the velocities due to each component wave. It is clear from Fig. 5 that after the instant of conjunction the lines of slack water and of strength of tide also move from their respective initial positions, and progress at varying rates of speed toward the positions proper for the next instant of conjunction. (See Fig. 6.)

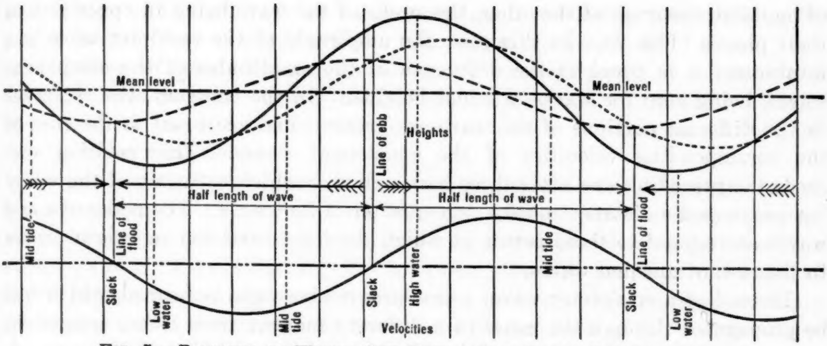


FIG. 5.—POSITION OF WAVES ONE HOUR AFTER THEIR CONJUNCTION.

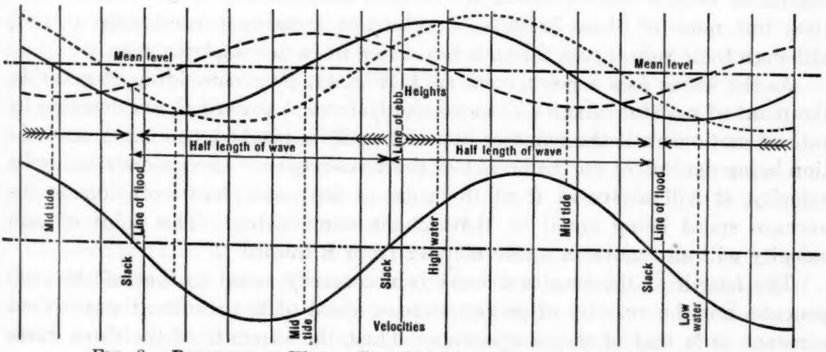


FIG. 6.—POSITION OF WAVES TWO HOURS AFTER THEIR CONJUNCTION.

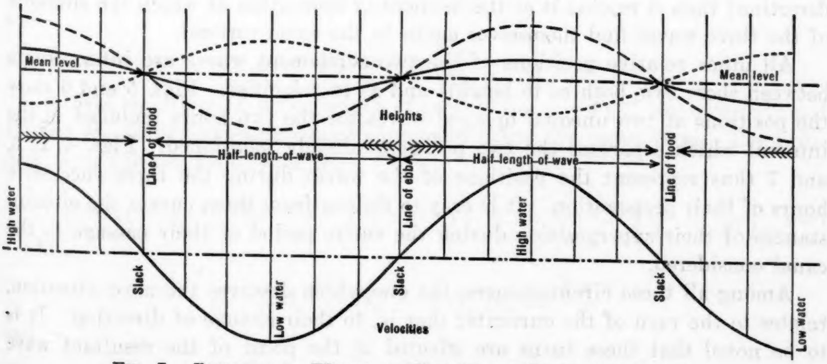


FIG. 7.—POSITION OF WAVES AT MOMENT OF THEIR OPPOSITION.

The second relative position of the waves which it is important to notice, is that which corresponds to the coincidence, in the same vertical of the trough of one with the crest of the other, the swells of the wave being in opposition in their phases (Fig. 7). In this case, the amplitude of the resultant wave is a minimum; it is equal to the difference of the amplitudes of the component waves, being zero for waves of equal heights. On the contrary, the velocities in the different portions of the wave are greatest and are equal to the sum of the corresponding velocities of the component waves. This position succeeds the preceding one after three hours, during which each one of the waves has progressed a distance equal to one-quarter of its length. The nodes of equal waves correspond to those points at which the least variation in height occurs in the case of unequal waves.

Instead of a stationary wave, a new progressive wave is formed, which will be propagated through the canal in a different manner from either component wave. This progressive wave will have points of minimum heights corresponding to nodal points accompanied by maximum velocities, and points of maximum heights corresponding to summits accompanied by minimum velocities; but none of these heights or velocities remains permanently at zero, although their values pass through zero twice in each complete cycle.

As the waves pass a point, such as *A* (Fig. 4), after coinciding at mid-tide, the point of mid-tide stage will move slowly from *A* toward *C*, accelerating its rate of motion until the point of opposition, *B*, is reached, the exact acceleration being dependent on the height of the minor wave. Then, slowly losing its velocity, it will arrive at *C* at the time of the next phase coincidence, the average speed being equal to that of the components. The point of zero velocity will also move in a like manner from *B* toward *C*.

The length of the resultant wave is necessarily equal to that of the components, but the velocity of propagation or speed of the resultant wave is not constant as is that of the components. Thus, the summits of the three waves are in the same vertical at the moment of their conjunction. The resultant wave is then out-distanced by that component which is propagated in the same direction; then it rejoins it at the moment of opposition at which the summits of the three waves find themselves again in the same vertical.

All other relative positions of the two component waves are intermediate between these two, both as to heights and as to velocities. Figs. 5 and 6 show the positions of two unequal opposed waves for the two hours included in the interval which separates the two positions already mentioned. Figs. 4, 5, 6, and 7 thus represent the positions of the waves during the three successive hours of their propagation. It is easy to deduce from these curves the circumstances of their superposition during the entire period of their passage in the canal considered.

Among all these circumstances, the one which deserves the most attention, relates to the turn of the currents; that is, to their change of direction. It is to be noted that these turns are effected at the point of the resultant wave located on the vertical where the surfaces of the component waves meet. Furthermore, when this point is found in that portion of the resultant wave corresponding to the falling tide, the currents diverge or separate from that

point which marks what is termed the "line of separation" of the currents, or "line of ebb". They approach or meet when the point of intersection occurs on that portion of the resultant wave corresponding to the rising tide, marking the line termed the line of meeting of the tide, or the "line of flood".

The lines of separation or of meeting are equally spaced at a distance of one-half wave length. Upon these lines the currents turn; their velocity increases with the distance from these lines, and it is a maximum at the half-way point between them.

In the resultant wave, the currents turn at half-tide at A, B, C, D, and E, and they reach their strength at the high and at the low water at the moment when the component waves are superposed at B and D and when in opposition at A, C, and E (Figs. 4 and 7). For all other points in the canal, the turns of the currents occur a greater or less time after half-tide, depending on the hour, or the position of the component waves, and the strength of the current comes one-quarter of the tidal period later. When the waves are equal, the currents turn simultaneously throughout the entire canal.

Therefore, if the water of the infinitely long canal is subjected to the action of two unequal and opposed waves, its surface will present the appearance of transmitting a wave having a direction and average speed equal to that of the major wave, but which will continually vary in speed and amplitude as it progresses. Furthermore, its velocity in any given vertical will not be in direct proportion to the amplitude at that point. The phenomenon of the interference of two opposed waves, having the same horizontal amplitude, the same speed of propagation, and the same period, but of different vertical amplitude, lends itself easily to mathematical demonstration.*

Assuming that, at the instant $t = 0$, the two profiles of the waves pass through the origin of co-ordinates taken upon the mean level, with the axis of x in that level, and that they correspond to the conjunction of the two waves which would accordingly be in the same phase—the equations of the profiles of the two waves would be:

$$y_1 = H_a \sin 2\pi \left(\frac{x}{L} - \frac{t}{T}\right)$$

and,

$$y_2 = H_b \sin 2\pi \left(\frac{x}{L} + \frac{t}{T}\right).$$

In these equations, L is the length of each of the waves; T, their period; and H_a and H_b, their semi-amplitudes above and below the mean level. The ordinate, y, of the resultant wave is:

$$y = y_1 + y_2 = H_a \sin 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) + H_b \sin 2\pi \left(\frac{x}{L} + \frac{t}{T}\right) \qquad (38)$$

Let,

$$K^2 = H_a^2 + H_b^2 + 2\, H_a H_b \cos \frac{4\pi t}{T}$$

and,

$$\tan \frac{2\pi\phi}{L} = \frac{H_a - H_b}{H_a + H_b} \tan \frac{2\pi t}{T} \qquad (39)$$

* "Hydraulique," Flamant, Paris, 1923, p. 486.

then Equation (38) can be written in the form:

$$y = K \sin \frac{2\pi}{L}(x - \phi) \qquad (40)$$

It follows from Equation (40) that at every point the maximum amplitude of oscillation has a special value, dependent upon the position of the point, and that it is comprised between $H_a + H_b$ and $H_a - H_b$. The maximum amplitude, $H_a + H_b$, is found at intervals equal to L, and the minimum amplitude, $H_a - H_b$, at the middle point of these intervals. At a given point, the duration of a complete oscillation is always equal to T. Likewise, at a given instant, the total length of the undulation is always equal to L; its mean speed of propagation is, therefore, equal to that of each of the component waves, but the speed varies with time. Differentiating Equation (39):

$$\frac{d\phi}{dt} = \frac{\frac{L}{T}}{\frac{H_a - H_b}{H_a + H_b}\sin^2\frac{2\pi t}{T} + \frac{H_a + H_b}{H_a - H_b}\cos^2\frac{2\pi t}{T}} \qquad (41)$$

which is a minimum for $t = 0$, $t = T$, etc., and a maximum for $t = \frac{T}{2}$, $t = \frac{3T}{2}$, etc.

The meeting points of the component waves and the positions of the lines of flux are found from the equality, $y_1 = y_2$; whence, $H_a \sin 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) = H_b \sin 2\pi\left(\frac{x}{L} + \frac{t}{T}\right)$, which gives,

$$\tan 2\pi\frac{x}{L} = \frac{H_a + H_b}{H_a - H_b}\tan 2\pi\frac{t}{T} \qquad (42)$$

Equation (42) shows that the lines are spaced at one-half wave length apart. The speed of their displacement is given by the expression,

$$\frac{dx}{dt} = \frac{\frac{L}{T}}{\frac{H_a - H_b}{H_a + H_b}\cos^2\frac{2\pi t}{T} + \frac{H_a + H_b}{H_a - H_b}\sin^2\frac{2\pi t}{T}} \qquad (43)$$

which is minimum for $t = \frac{T}{2}$, $t = \frac{3T}{2}$, etc., and a maximum for $t = 0$, $t = T$, $t = 2T$, etc.

When the two waves have the same vertical amplitude, $H_a = H_b$, the resultant motion is the "chop". Its special laws which result from the preceding equations are summarized as follows:

1.—At the instant of conjunction, the velocities are annulled, or slack-water prevails for the entire length of the canal, and the amplitude of the resultant wave is double that of the component waves.

2.—At the instant of opposition, the velocities reach their maximum in all that part of the canal where the mean level prevails. There is no change of level produced, during the entire tide, at those points corresponding to the summits of the waves in opposition, and the velocity there is greater than elsewhere.

3.—The positions of the lines of flux and of reflux remain invariable for the entire tide; they coincide with that of the summits of the waves in conjunction.

General Remarks.—It must be recognized that theory, even if perfect, would still remain inadequate to solve completely the complex problems which the regimen of the tide raises in practice. The ideal canal is not found in Nature; waterways have a more or less irregular bed, and almost always they have a length less than that of the tidal wave which traverses it. This latter fact alone has hitherto sufficed to upset all theories, because the tidal wave cannot be propagated in them as has been assumed for an infinitely long canal. In this hypothetical canal the waves always travel forward, without mutually disturbing each other; each one before and after its passage finds and leaves any point in its original condition; they then disappear without leaving any trace of their existence. It is not so when the length of the canal is limited; in introducing itself the wave encounters the effects of preceding waves; the circumstances of its propagation, consequently, are modified by reason of these effects, which will be found impossible to follow in all their consequences. A complex and variable regimen results from all this, according to circumstances, controlled by the length and characteristics of the bed, which it seems must forever escape satisfactory analysis.

On the other hand, by a consideration of the theoretical canal, it is possible to treat some of the influences which affect the tidal wave, and to forecast what general effect may be expected from variations of length, width, depth, and the material of the bed of the canal. By means of this study it is possible to determine relative if not absolute values of such effects. It is in this spirit that the problem is approached in the following pages.

Propagation of the Tide in a Canal Communicating with an Infinitely Large Reservoir

Insufficiency of Theory That Flow Caused by Surface Slope Accounts for Tidal Flow.—Assume a horizontal canal of constant cross-section communicating at one of its ends with the sea, and at the other with a reservoir sufficiently large to be considered as infinite; that is, so large that its level remains constant. Let $A\,B$ in Fig. 8 be the mean level of the sea assumed to be the same as that of $B\,C$ in the hypothetical reservoir, and A_2, A_4, the extreme levels attained by the tide. Assume that, at a given instant, a state of equilibrium of the entire liquid mass is established, the level of the sea being at A, and the surface of the water horizontal for the entire length of the canal. Suppose the level then rises to A_1. If it should remain immobile for a sufficiently long time at this point, a permanent motion would be set up in the canal between A and B, defined by the equations of constant uniform flow. The free surface between these points would take the form of a curve tangent at A_1 to the horizontal, and convex upward.

However, things do not happen in this manner. The level is constantly variable and the permanent regimen, corresponding to any such height as A_1, never has time to become established. Even the state of equilibrium which was assumed when the level was at A, at the height, $B\,C$, would never be realized unless the level should remain motionless at A for a sufficiently long time.

During the period when the level rises from A toward A_2, it reaches some height A_1, before the water entering through the mouth can have filled completely the triangle, AA_1B. It rises relatively faster at A than at any other intermediate point between A and B, and the curve of level takes a concave form upward, such as that shown by the broken line in Fig. 8.

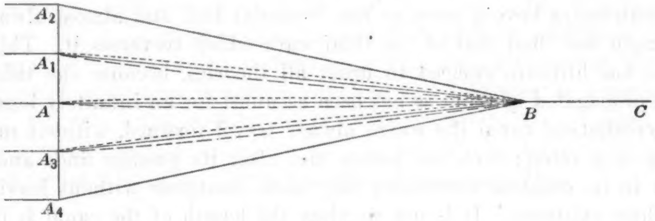


FIG. 8.

On the other hand, when the level falls from A_2 toward A, as it is continually variable, it leaves any position before the corresponding permanent state can become established in the canal, and the curve of level is then convex upward, as is shown by dotted lines. The convexity of this curve will be even greater than that which would correspond to a permanent regimen.

The same effects will be produced in the portion of the oscillation that takes place below the level of equilibrium, between A and A_4. When the level comes to A_3 in falling, the surface of the water in the canal is convex upward and, conversely, it is concave when the level passes through A_3 in rising. The curves in broken lines then represent the levels during the flood, and those in dotted lines, the levels during the ebb.

Under these conditions, the velocities at each point of the canal will be nil because of reversal at the moment of half-tide, when the level passes through the mean level; they will be a maximum, whether on the flood or the ebb, at the moment the difference of level is greatest.

The phenomenon is in reality much less simple than the preceding description might indicate. The tidal wave is not transmitted instantaneously in the entire length of the canal, so that the high waters, the slack waters, and the low waters are not simultaneous in the entire stretch. They are successive because of the retardation of the propagation of the wave. The speed of this propagation itself is different from what it would be in a canal of constant depth. If the level of the sea is considered to be held motionless during a sufficient time at a height, A_1, and the corresponding permanent regimen is established with a certain mean velocity of flow, U_1, an undulation created at A_1 will be propagated toward B with a speed greater by U_1, approximately, than that which would correspond to the depth. On the other hand, the tidal wave entering into the canal possesses a certain energy which, except for the losses due to friction, remains constant. Because the depth diminishes from A_2 toward B an increase of the velocities results from the constancy of this energy, which again has for a consequence an increase of the speed of propagation. For this reason, the speed of propagation of the wave and the velocities of the water, will be greater in the following sections than they would be in a

horizontal canal with free surface and of the same depth as this section; and the difference will go on increasing as the point, *B,* is approached. (From the origin of the canal toward the point, *A,* this speed of propagation and the velocities will have adjusted themselves with regard to local circumstances and to the depth corresponding to the level, A_2.)

Influence of an Infinitely Large Reservoir.—What influence is exerted upon the wave of translation by the intervention at any point of an infinitely large reservoir? This hypothetical body of water has been defined as one sufficiently large to be unaffected by the emptying of the volume of the tidal wave into it. Consequently, its surface remains at a constant level, which is also assumed to be the mean level of the tidal sea and of the canal.

Equation (20) shows that no matter what the energy per unit length of the wave may be, the square of its height will vary inversely with the width of the canal, l. It follows from this and from the demonstration given under the heading, "Influence of Change of Width" (see Fig. 3), that if the width of the canal is made very large, h will become very small. Since the reservoir constitutes an infinite expansion in the width of the canal, l becomes very large, approaching infinity, and h becomes correspondingly small, approaching zero. Therefore, the wave of translation, whether positive or negative, must disappear at the outlet of the canal into the reservoir. Observation reveals that not only does the wave disappear at the outlet, but that the influence of the reservoir in decreasing the height of the wave is reflected back for the entire length of the canal, causing both heights and velocities at all points in the canal to differ from their theoretical values.

Explanation.—To find an explanation of this phenomenon, it is desirable to revert to the theory of the propagation of the tide in a canal as being formed by a series of elementary waves of translation, or successive fluxes and refluxes. Neglecting friction, the first flux being created on the rising tide would propagate itself indefinitely without change, until it reached the reservoir, where it must disappear. The flux is propagated at a definite speed through the canal, and a certain period is required to elapse from the time it appears at the entrance of the canal until it disappears at the outlet. Why does not the flux manifest itself simultaneously throughout the canal immediately upon its appearance? The answer, of course, is that the front or anterior portion of the flux must successively overcome the friction of the banks and the resistance or inertia opposed to its propagation by each particle of water in the canal. The height and the form of the flux are moulded by these resistances. The decrease of these resistances of inertia and the sudden cessation of all resistance to the flux upon entering into the reservoir permit a more rapid speed of propagation between the entrance and the outlet; but even this more rapid speed is never instantaneous, and a certain time would be required for the surface of the flux to acquire the slope it would assume if it were allowed time to establish the conditions of permanent flow. The flux is accompanied by a given velocity of flow of the water in the canal to supply the intumescence. The cessation of the forces of inertia opposing the flux then results in an increase of this velocity of flow.

Now, if a piston that just fits the cross-section of the canal were inserted at any point and given a push in a direction to increase the volume already flowing under the influence of the flux, the effect of this applied energy would be to generate a negative wave propagating itself in the direction opposite to that of the flux. This would be in addition to the positive wave propagated in the same direction, which for the time being is neglected.

The cessation of the resistances of inertia at the outlet will have the same effect as the piston or the addition of energy as assumed, thus creating a negative flux at the outlet which is propagated back through the canal toward the entrance.

The height of the negative flux (or depression) at the outlet will be exactly equal to that of the flux, so that their sum shall be zero, and the resultant velocities of the two fluxes, being in the same direction, will add their effects. Hence, the final velocity will be twice the theoretical, which is the velocity that would correspond to the computed height of the wave of translation for the point in question under the assumption of an infinitely long canal.

Source of the Energy Generating the Negative Flux.—The energy of the wave of translation is reduced only by friction, and in the infinitely long canal it will consist of equal parts of potential energy represented by the height of the wave and of kinetic energy as represented by the velocity of the water. Under proper conditions one kind of energy is convertible into the other; yet the total sum of available energy at any given point is constant.

The hypothetical reservoir is of sufficient size to absorb all the energy of the wave, with no effect upon it, and the energy represented by velocity of the water is expended at the outlet in the whirls, boils, and eddies that usually occur when flowing water is brought to rest in still water. If no change had occurred in the flux until it had reached the reservoir, one-half its energy, (kinetic), would have been so expended. The other half, the potential, must be converted into kinetic energy before it can be dissipated in this manner, and it is this energy being converted into kinetic energy—manifested by increased velocities of the water—which generates the opposed negative flux.

Resultant Effect.—Each flux following the first must be subject to the same requirement of disappearance in the reservoir, and must generate an opposite negative flux. Hence, it follows that, as the complete wave of translation is being propagated toward the reservoir, it is being continually modified by an opposed negative wave propagated from the reservoir toward the sea, and generated by the influence of the reservoir upon the original wave.

A similar reasoning would show that the negative wave of translation propagated from the sea toward the reservoir, and which may be said to absorb energy from the water into which it is propagated, will, when reaching the reservoir, cause an opposite positive wave to be generated there which will be propagated toward the sea.

A rigorous application of this reasoning would lead to the conclusion that the propagation of a single positive wave through the canal would result in an infinite series of alternating positive and negative waves being set up. Each positive wave from the sea would generate a corresponding negative wave from

the reservoir, and *vice versa.* Under the influence of friction, however, each succeeding wave would become smaller and smaller, so that the series would finally die out, for all practical purposes. The resultant profile of the canal at any instant would be the algebraic sum of all the wave heights at that instant, and the resultant velocity would be the sum of all the velocities.

Formula for the Variation of the Tide at a Given Point.—At a given point the cycle of variation of any wave surface, being a result of the great ocean tidal wave, will be expressed approximately by a sinusoidal law, and its trace may be expressed by the equation,

$$Y = H \cos 2\,\pi \left(\frac{t - t_x}{T}\right) \dots\dots\dots\dots\dots (44)$$

Each point of the canal will have a different set of values for H and t_x. It is likewise manifest that values of H and t_x for the same point will be different for the positive and negative waves.

Changing the point of origin for counting the time, t_x, and computing the time, t_y, for the propagation of a wave from the reservoir toward the sea, the same formulas can be used to compute the variation of the reflected wave from the reservoir to the sea, the proper constants being inserted in the general equations.

Let t_L be the time required for the positive wave to be transmitted from the sea to the reservoir, and t_y the time required for the reflected negative wave to be transmitted to the point represented by a given set of values of H and t_x (the factor, $- H'$, being the corresponding value of the height of the reflected wave at that point), the following is the equation of the variation of the reflected wave at the same point:

$$- Y' = - H' \cos 2\,\pi \left(\frac{t - (t_L + t_y)}{T}\right) \dots\dots\dots (45)$$

Adding Equations (44) and (45), the actual height, h, of the tide at the selected point at the time, t, is,

$$h = Y - Y' = H \cos 2\,\pi \left(\frac{t - t_x}{T}\right) - H' \cos 2\,\pi \left(\frac{t - (t_L + t_y)}{T}\right) \dots (46)$$

This equation may be written in the shorter form,

$$h = \sqrt{H^2 + H'^2 - 2\,H\,H' \cos 2\,\pi \left(\frac{t_x - (t_L + t_y)}{T}\right)} \cos 2\,\pi \left(\frac{t - t_m}{T}\right) . (47)$$

in which, t_m may be found from the equation,

$$\tan 2\,\pi \frac{t_m}{T} = \frac{H \sin 2\,\pi \dfrac{t_x}{T} - H' \sin 2\,\pi \left(\dfrac{t_L + t_y}{T}\right)}{H \cos 2\,\pi \dfrac{t_x}{T} - H' \cos 2\,\pi \left(\dfrac{t_L + t_y}{T}\right)} \dots\dots (48)$$

It will be observed that t_m is that value of t for which h will be a maximum. It, therefore, represents the time required for the resultant wave to be propagated from the sea to the point considered.

It is evident from the preceding that for any number of reflected waves that may be considered, a general expression for the resultant wave may be written as follows:

$$h = H_1 \cos 2\,\pi \left(\frac{t - t_1}{T}\right) + H_2 \cos 2\,\pi \left(\frac{t - t_2}{T}\right)$$

$$+ H_3 \cos 2\,\pi \left(\frac{t - t_3}{T}\right) + \ldots + H_n \cos 2\,\pi \left(\frac{t - t_n}{T}\right) \ldots\ldots (49)$$

When any two waves simultaneously affect the same point and their heights are additive because they have the same sign, the resultant velocity must be the difference between the component velocities. A general formula, similar to Equation (49), may be written for the resultant velocity at the same point, as follows:

$$V = V_1 \cos 2\,\pi \left(\frac{t - t_1}{T}\right) - V_2 \cos 2\,\pi \left(\frac{t - t_2}{T}\right)$$

$$- V_3 \cos 2\,\pi \left(\frac{t - t_3}{T}\right) - \ldots - V_n \cos 2\,\pi \left(\frac{t - t_n}{T}\right) \ldots\ldots (50)$$

in which, V_1, V_2, V_3, etc., are found directly from the corresponding values of H_1, H_2, H_3, etc., by means of the equation,

$$V_n = H_n \sqrt{\frac{g}{D + h}} \ldots\ldots (51)$$

the value of h being that given by Equation (49).

Equation (49) may be made to represent conditions at the outlet of the canal into the reservoir by making substitutions, as follows: $H_2 = -H_1$; $H_4 = -H_3$; $H_n = -H_{n-1}$; $t_1 = t_2$; $t_3 = t_4$; and $t_{n-1} = t_n$. Likewise, Equation (50) may be written so as to represent velocities at the same point by substituting $V_2 = -V_1$; $V_4 = -V_3$; and $V_n = -V_{n-1}$. At the sea entrance to the canal, $H_1 = H_0$; $H_3 = -H_2$; $H_5 = -H_4$; $H_n = -H_{n-1}$; $t_1 = 0$; $t_2 = t_3$; $t_1 = t$; $t_n + 1 = t_n$; $V_1 = V_0$; $V_3 = -V_2$; $V_5 = -V_4$; and $V_n = -V_{n-1}$.

In a similar manner Equations (49) and (50) may be applied to all other points in a tidal canal opening into a tideless reservoir by changing signs in alternate (second, fourth, sixth, etc.) quantities. Thus, in Equation (49), change the sign from plus to minus in alternate quantities, in which n is even. Conversely, under the same circumstances change the sign from minus to plus for alternate quantities in Equation (50).

To determine the values of t for which these values of h and v are maxima, differentiate the equations with reference to t, place the first differential equal to zero, and solve for t, the particular value of which will be designated t_m:

$$\frac{dh}{dt} = - H_1 \sin 2\,\pi \left(\frac{t - t_1}{T}\right) + H_2 \sin 2\,\pi \left(\frac{t - t_2}{T}\right)$$

$$- H_3 \sin 2\,\pi \left(\frac{t - t_3}{T}\right) \ldots \mp H_n \sin 2\,\pi \left(\frac{t - t_n}{T}\right) = 0 \ldots\ldots (52)$$

whence, $\tan 2\pi \frac{t_m}{T}$ equals,

$$\frac{H_1 \sin 2\pi \frac{t_1}{T} - H_2 \sin 2\pi \frac{t_2}{T} + H_3 \sin 2\pi \frac{t_3}{T} \dots \pm H_n \sin 2\pi \frac{t_n}{T}}{H_1 \cos 2\pi \frac{t_1}{T} - H_2 \cos 2\pi \frac{t_2}{T} + H_3 \cos 2\pi \frac{t_3}{T} \dots \pm H_n \cos 2\pi \frac{t_n}{T}} \quad (53)$$

Similarly, for t_{mv}, the time of maximum velocity, $\tan 2\pi \frac{t_{mv}}{T}$ equals,

$$\frac{V_1 \sin 2\pi \frac{t_1}{T} + V_2 \sin 2\pi \frac{t_2}{T} - V_3 \sin 2\pi \frac{t_3}{T} \dots \mp V_n \sin 2\pi \frac{t_n}{T}}{V_1 \cos 2\pi \frac{t_1}{T} + V_2 \cos 2\pi \frac{t_2}{T} - V_3 \cos 2\pi \frac{t_3}{T} \dots \mp V_n \cos 2\pi \frac{t_n}{T}} \quad (54)$$

It is quite evident that the value of t_m for heights is not the same as that of t_{mv} for velocities, and, therefore, the time of maximum velocities will not coincide with the time of maximum or of zero heights.

After $-V_{n-1}$ has been substituted for V_n in Equation (50) it is possible to evaluate the frictional coefficient, F, in an easy and rapid manner. Combining it with Equation (51):

$$V = 2\, V_1 \cos 2\pi \left(\frac{t - t_1}{T}\right) = 2\, H_1 \sqrt{\frac{g}{D + h}} \cdot \cos 2\pi \left(\frac{t - t_1}{T}\right) \dots (55)$$

Whence,

$$H_1 = \frac{1}{2} V_0 \sqrt{\frac{D}{g}} \dots\dots (56)$$

in which, V_0 equals the maximum value of V occurring when $t = t_1$. The value of V_0 can be measured readily at the outlet by any of the usual methods of determining the maximum mean velocity, and this value when determined enables H_1 to be found from Equation (56). Then when H_1 is substituted in Equation (26), in which all factors except F will be known, the frictional coefficient for the particular case will be given. Bonnet found this value to be 2.48 for the Scheldt River (0.51 English measure). The writer has found 3.05 (or 0.60 English measure) to be the usual value for canals of 6 to 12 ft. in depth, while Bonnet's value appears to apply to larger canals, such as the Cape Cod and the Suez Canals.

Since the direct and reflected waves at the outlet have all their ordinates exactly equal and of opposite sign, thus completely annulling each other, it is apparent that they are in opposition at that point, and that the time of maximum velocity corresponds to the time when the maximum ordinates are in opposition. The instant of this occurrence, therefore, measured from the time when the wave entered into the canal at its mouth, gives the time, t_1, consumed by the wave in propagating itself from the mouth to the outlet, and this time can also be determined by observation.

In Equation (50) change the signs of the alternate second, fourth, sixth, etc., quantities from negative to positive and divide by $\sqrt{\frac{g}{D + h}}$. To the

result add Equation (49) with the signs of alternate second, fourth, sixth, etc., quantities changed from positive to negative. The resulting equation, simplified, takes the form,

$$h + v \sqrt{\frac{D + h}{g}} = 2\ H_1 \cos 2\ \pi \left(\frac{t - t_1}{T}\right) \ldots\ldots\ldots\ldots (57)$$

Now, $v \sqrt{\frac{D + h}{g}}$ may be considered as the height of a simple wave of translation which alone would give rise to a velocity, v; it will be designated the velocity height and represented by h_v.

Equation (57) then takes the form,

$$h + h_v = 2\ H_1 \cos 2\ \pi \left(\frac{t - t_1}{T}\right) \ldots\ldots\ldots\ldots (58)$$

Equation (58) is general, and will be true for any moment, at any point on the canal. The conclusion is drawn from it that in a uniform canal communicating with a tidal sea at one end, and with a tideless basin at the other, the actual height of the water in the canal at any instant plus a height equivalent to the velocity of the water at the same instant will be equal to double the height of the original wave of translation as it would have been transmitted at the same instant in a canal of infinite length; that is, in the hypothetical canal as defined in this paper.

Similarly, if Equation (50) is multiplied by $\sqrt{\frac{g}{D + h}}$ and added to Equation (49) as before (with alternate signs changed), the resulting equation takes the form:

$$v + \frac{\sqrt{g}\ h}{\sqrt{D + h}} = 2\ V_1 \cos 2\ \pi \left(\frac{t - t_1}{T}\right) \ldots\ldots\ldots\ldots (59)$$

In Equation (59), $\frac{\sqrt{g}\ h}{\sqrt{D + h}}$ equals a velocity which would be due to the height, h, in an infinitely long canal of the same dimensions. Denoting this quantity by v_h,

$$v + v_h = 2\ V_1 \cos 2\ \pi \left(\frac{t - t_1}{T}\right) \ldots\ldots\ldots\ldots (60)$$

From Equation (60) it is conclusive that the velocity at any point in the canal at any instant, added to the velocity which would accompany—in an infinitely long canal—a wave of the same height as is found at the same instant, the sum will be equal to twice the velocity which the wave would have generated at that instant in the hypothetical canal.

Since the second members of Equations (58) and (60) are constant for any point at a particular instant, it shows that any decrease in the height of water above or below the theoretical height for the long canal must be accompanied by an increase in velocity, or *vice versa*, and that the total amount of energy manifested is the same in any case.

For the compound or resultant wave there is no approximately constant value for the ratio between h and v, as in the case of the simple wave of

translation. Failure to comprehend this departure of the compound wave from the laws pertaining to the simple wave has heretofore made the computation of the tidal characteristics of any given point in a tidal canal quite unsatisfactory. It is now proposed to show that a satisfactory agreement between observed tidal phenomena and the computed values of such phenomena can be reached, and that the computation in any case can be made sufficiently close to enable all tidal phenomena in canals of fairly uniform size to be predicted.

Application of the Theory to the Suez Canal

That section of the Suez Canal between the Red Sea and the Bitter Lakes is an example of the type of canal under consideration. It is 26 km. (16 miles) long, and has an average semi-amplitude of tide of 2.24 ft. at the Suez entrance. The amplitude gradually diminishes until at the Bitter Lakes all tidal variation disappears. The Bitter Lakes have an area of about 48 500 acres, which is large enough to receive the tidal flow without appreciable change in level.

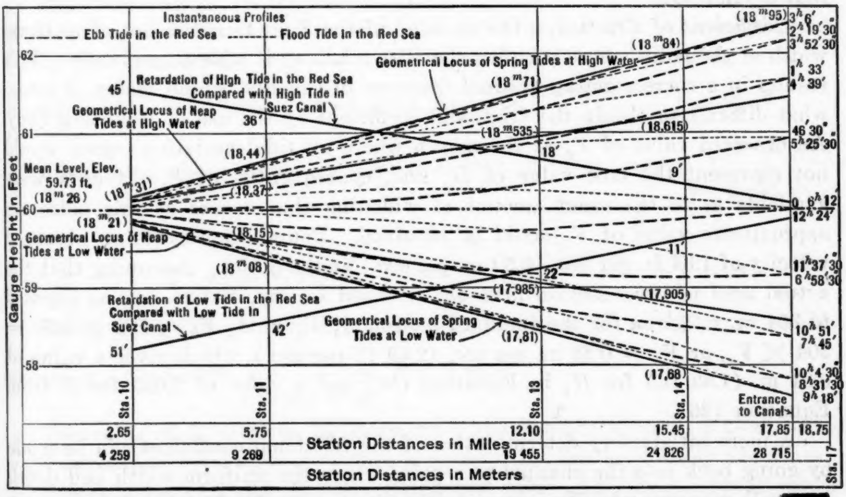


Fig. 9.—Graphical Representation of the Mean Instantaneous Profiles of the Suez Canal Between the Red Sea and the Bitter Lakes: Third Quarter of 1898.

Lévy tested his theory* by comparison with observations on the Suez Canal. He found that while it could account for the observed heights of the tide, it could not account for the observed velocities, particularly those at the outlet of the canal into the Bitter Lakes. This canal was also the subject of rather complete tidal observations, some of the results of which are given by Bourdelles† in an idealized graphical representation of the mean instantaneous profiles of the canal between the Red Sea and the Bitter Lakes (Fig. 9).

* "Leçons sur la Théorie des Marées," par Maurice Lévy, Gauthier-Villars, 1898.

† "Etude sur le Régime de la Marée dans le Canal de Suez," *Annales des Ponts et Chaussees*, 1898, 3me trimestre.

In his memoir, Bourdelles directed attention to the failure of any previous theory to account for the high velocities at the outlet where all existing theories called for the disappearance of the velocity when the height of the wave vanished. He found a satisfactory agreement with theory for velocity near the Suez end, but the discrepancies increased toward the lake end. To explain this, he was forced to resort to the theory that the excess velocity over that caused by the height of wave must be due to a flow under the influence of the surface slope of the wave, but no proof of such flow could be adduced.

Under the reflected wave theory, the computed velocities agree quite well with the observed velocities, and the computed heights are almost identical with those given by Bourdelles. Unfortunately, Bourdelles gives very little information as to velocities, simply naming the maximum velocities observed at certain points without giving the amplitude of the tide at the Suez Station during the tidal period which gave rise to those velocities. It is assumed that the velocities which he does give were generated during an average tide such as he indicates upon his profiles, and the comparisons are based upon that assumption.

Coefficient of Friction.—The channel of the Suez Canal is not of uniform width at the point where it enters the Bitter Lakes; it widens gradually. This results in a corresponding gradual decrease in velocity, which makes it somewhat difficult to obtain the frictional coefficient by the use of Equation (56). An observed value of V_0 at the mouth where all tidal variation ceases would not represent the true value of H_1 and, consequently, of F. If data were available as to the exact amount of widening that occurs at this point an approximate value of V_0 could be obtained. Bourdelles reports an observed velocity of 1.64 ft. per sec. (0.50 m. per sec.) at the outlet. Assuming that the actual area of cross-section here has widened to, say, 500 sq. m., as opposed to 304 sq. m. given for the standard section $A_1 V_1 = A_0 V_0 = 500 \times 0.50 = 304 \times V_0$, or $V_0 = 0.82$ m. per sec. (2.69 ft. per sec.), which gives a value of 0.31 m. (1.33 ft.) for H_1 in Equation (56) and a value of 2.655 for F from Equation (26).

A more satisfactory determination of the frictional coefficient can be made by going back into the channel to a point where the uniform width and depth are still maintained. For example, at Station 11 in Fig. 9, the tide and velocity of the current are both given. In this case it is necessary to proceed in a somewhat different manner, since the heights of the direct and reflected waves will not be equal at this point like they are at the outlet. It is now necessary to make use of Equation (57). In that equation, substitute t_m for t; v is not the maximum velocity in this case, but the velocity at high water, observed simultaneously with h_m.

At the time of the observations, the canal was 22 m. (72 ft.) wide on the bottom, with side slopes of 1 on 2. The depth was 8 m. (26 ft.) below the datum plane of mean tide, which was at an elevation of +18.26 m. (59.73 ft.). The following, therefore, are the basic data on which the computations are based:

	Metric units.	English units.
Area of cross-section at mean sea level, in square meters and square feet............	304	3 272
Width of canal at mean sea level, in meters and feet................................	54	177
Wetted perimeter, in meters and feet........	57.77	189.54
Semi-amplitude of the tide at Suez, in meters and feet................................	0.69	2.26
Length of canal, in kilometers and miles.....	26	1.6
Reduced depth, $\frac{304}{54}$, in meters and feet.......	5.63	18.47

For Chalouf, Bourdelles gives the following data: Height, 0.18 m. (0.59 ft.), and velocity, 0.80 m. per sec. (2.62 ft. per sec.).

As t_1 is the time required for the wave of translation to travel to the point of observation under the assumption of an infinitely long canal,

$$t_1 = \frac{x}{c} = \frac{x}{\sqrt{g\,(D + h)}} = \frac{19\,446}{\sqrt{(5.63 + 0.18)\,(98.06)}} = 2\,576 \text{ sec.} = 43 \text{ min.}$$

The other term, t_m, is the time at which high water occurred at the point of observation, which is given by Bourdelles as occurring 36 min. later than at Suez, or about 30 min. later than at the canal entrance, making $t_m - t_1 = (-43 + 30) = -13$ min., which gives the value, $2\,\pi\left(\frac{t_m - t_1}{T}\right) = 6°\,17'$.

Substituting these values in Equation (57), $0.18 + 0.80\sqrt{\frac{5.63 + 0.18}{9.806 +}} = 2\,H_1 \cos 6°\,17'$; whence, $H_1 = \dfrac{0.18 + 0.80\sqrt{\frac{5.63 + 0.18}{9.806 +}}}{2 \cos 6°\,17'} = 0.4023$.

Substituting this value in Equation (26), in which, $H_0 = 0.69$, $x = 19\,446$, and $D = 5.63$; $8.2439 = \dfrac{F\,p\,x}{2\,\rho\,l\,\sqrt{g}}$; $F = 2.4816$.

The value of F as given by Bonnet for the Scheldt River is 2.4802. The close agreement between these values is noteworthy, but is probably accidental.

Computation of Tidal Curves.—Using this value of F, the computations in Tables 1 and 2 may be made. These tables give the values of H and of t for various selected stations which are the points of observation for the data upon which Bourdelles' curves are based. In Table 1 the value of $\dfrac{F\,p}{2\,\rho\,l\,\sqrt{g}}$ equals 0.000423935. The data contained make it possible to write the following general equations of the tide for those stations; for example, at Station 3889 (Table 1), the equation for the positive wave is:

$$h = +\,0.622 \cos 2\,\pi\left(\frac{t - 496}{T}\right) - 0.159 \cos 2\,\pi\left(\frac{t - 6\,298}{T}\right)$$
$$+\,0.123 \cos 2\,\pi\left(\frac{t - 7\,290}{T}\right) - 0.01 \cos 2\,\pi\left(\frac{t - 13\,092}{T}\right) \ldots\ldots \quad (61)$$

TABLE 1.—COMPUTATIONS FOR DETERMINING THE GENERAL EQUATIONS FOR POSITIVE WAVES, THE SUEZ CANAL.

H, in meters.	$D^{3/2} \log_e H$.	$\frac{3}{2} D^{1/2} H$.	$\frac{3 H^2}{16 D^{1/2}}$.	$-\frac{F p \Delta x}{2 \rho l \sqrt{g}} + K$.	$\frac{F p \Delta x}{2 \rho l \sqrt{g}}$	Δx*, in meters.	Cumulative distance, in meters.	$D + h$, in meters.	RATE OF PROPAGATION, IN METERS PER SECOND. At station.	RATE OF PROPAGATION, IN METERS PER SECOND. Mean.†	Time of propagation, in seconds.	Cumulative time, in seconds.	Equivalent phase, in degrees.
						FIRST COMPONENT WAVE, POSITIVE.							
+0.690	− 4.9569	+2.4557	+0.0376	− 2.4636	0.	0	0	6.52	7.8727	......	0	0	0
+0.650	− 5.7543	+2.3134	+0.0333	− 3.4076	0.9440	2 226	2 226	....	......	......	....	......	......
+0.622	....	....	....	....	....	......	3 889	6.21	7.8038	7.8382	496	496	3.99
+0.600	− 6.8230	+2.1354	+0.0284	− 4.6602	1.2526	2 955	5 181	....	......	......	....	......	......
+0.550	− 7.9864	+1.9575	+0.0239	− 6.0050	1.3448	3 172	8 353	....	......	......	....	......	......
+0.537	....	....	....	....	....	......	9 260	6.07	7.7153	7.7595	692	1 188	9.56
+0.500	− 9.2594	+1.7796	+0.0198	− 7.4600	1.4550	3 432	11 785	....	......	......	....	......	......
+0.467	....	....	....	....	....	......	14 260	5.94	7.6323	7.6738	652	1 840	14.81
+0.450	−10.6659	+1.6015	+0.0160	− 9.0484	1.5884	3 747	15 532	....	......	......	....	......	......
+0.403	....	....	....	....	....	......	19 446	5.81	7.5483	7.5903	683	2 523	20.31
+0.400	−12.2394	+1.4236	+0.0126	−10.8032	1.7548	4 139	19 671	....	......	......	....	......	......
+0.350	−14.0225	+1.2456	+0.0097	−12.7672	1.9640	4 633	24 304	....	......	......	....	......	......
+0.348	....	....	....	....	....	......	24 456	5.68	7.4633	7.5058	667	3 190	25.68
+0.333	−14.6880	+1.1852	+0.0088	−13.4940	0.7268	1 714	26 018	5.63	7.4303	7.4468	207	3 397	27.35
						SECOND COMPONENT WAVE, NEGATIVE.							
−0.333	−14.6880	−1.1852	+0.0088	−15.8644	....	......	26 018	....	......	......	....	3 397	27.35
−0.316	....	....	....	....	....	......	27 544	....	......	......	207	3 604	29.02
−0.300	−16.0822	−1.0677	+0.0071	−17.1428	1.2784	3 015	29 033	....	......	......	....	......	......
−0.267	....	....	....	....	....	......	32 554	....	......	......	667	4 271	34.39
−0.250	−18.5175	−0.8897	+0.0049	−19.4023	2.2595	5 330	34 363	....	......	......	....	......	......
−0.225	....	....	....	....	....	......	37 740	....	......	......	683	4 954	39.89
−0.200	−21.4970	−0.7118	+0.0032	−22.2056	2.8033	6 612	40 975	....	......	......	....	......	......
−0.190	....	....	....	....	....	......	42 740	....	......	......	652	5 606	45.14
−0.159	....	....	....	....	....	......	48 111	....	......	......	692	6 298	50.71
−0.150	−25.3403	−0.5339	+0.0018	−25.8724	3.6668	8 649	49 624	....	......	......	....	......	......
−0.139	−26.3579	−0.4835	+0.0015	−26.8399	0.9675	2 283	51 907	....	......	......	....	......	......
−0.139	....	....	....	....	....	......	52·000	....	......	......	496	6 794	54.70
						THIRD COMPONENT WAVE, POSITIVE.							
+0.139	−26.3579	+0.4835	+0.0015	−25.8729	....	......	52 000	....	......	......	....	6 794	54.70
+0 123	....	....	....	....	....	......	55 889	....	......	......	496	7 290	58.69
+0.120	−28.3217	+0.4271	+0.0011	−27.8935	2.0206	4 766	56 673	....	......	......	....	......	......
+0.105	....	....	....	....	....	......	61 260	....	......	......	692	7 982	64.26
+0.100	−30.7571	+0.3559	+0.0008	−30.4004	2.5069	5 913	62 586	....	......	......	....	......	......
+0.089	....	....	....	....	....	......	66 260	....	......	......	652	8 634	69.51
+0.085	−32.9270	+0.3025	+0.0006	−32.6239	2.2235	5 245	67 831	....	......	......	....	......	......
+0.076	....	....	....	....	....	......	71 446	....	......	......	683	9 317	75.01
+0.070	−35.5213	+0.2491	+0.0004	−35.2718	2.6479	6 246	74 077	....	......	......	....	......	......
+0.065	....	....	....	....	....	......	76 456	....	......	......	667	9 984	80.38
+0.062	−37.1425	+0.2207	+0.0003	−36.9215	1.6497	3 891	77 968	....	......	......	....	......	......
+0.062	....	....	....	....	....	......	78 000	....	......	......	207	10 191	82.05
						FOURTH COMPONENT WAVE, NEGATIVE.‡							
−0.062	....	....	....	....	....	......	78 000	....	......	......	....	10 191	82.05
−0.050	....	....	....	....	....	......	79 544	....	......	......	207	10 398	83.72
−0.040	....	....	....	....	....	......	84 554	....	......	......	667	11 065	89.09
−0.030	....	....	....	....	....	......	89 740	....	......	......	683	11 748	94.59
−0.020	....	....	....	....	....	......	94 740	....	......	......	652	12 400	99.83
−0.010	....	....	....	....	....	......	100 111	....	......	......	692	13 092	105.41
−0.000	....	....	....	....	....	......	104 000	....	......	......	496	13 588	109.40

* Δx is the distance between stations corresponding to selected values of H.

† These figures are the means of those in the preceding column.

‡ As this wave is very small, it is assumed to become extinguished progressively in this reflection.

TABLE 2.—COMPUTATIONS FOR DETERMINING THE GENERAL EQUATIONS FOR NEGATIVE WAVES, THE SUEZ CANAL.

H, in meters.	$D^{3/2} \log_e H$.	$\frac{3}{2} D^{1/2} H$.	$\frac{3 H^2}{16 D^{1/2}}$	$-\frac{F p \Delta x}{2 \rho l \sqrt{g}} + K$.	$\frac{F p \Delta x}{2 \rho l \sqrt{g}}$.	Δx*, in meters.	Cumulative distance, in meters.	$D + h$, in meters.	Rate of Propagation, in Meters per Second. At station.	Mean.†	Time of propagation, in seconds.	Cumulative time, in seconds.	Equivalent phase, in degrees.
First Component Wave, Negative.													
−0.690	− 4.9569	−2.4557	+0.0376	− 7.3750	0.	0	0	4.94	6.9601	......	0	0	......
−0.650	− 5.7543	−2.3134	+0.0333	− 8.0344	0.6594	1 555	1 555	....	......	......	....	......	......
−0.600	− 6.8230	−2.1354	+0.0284	− 8.9300	0.8956	2 113	3 668	....	......	......	....	......	......
−0.595	....	....	....	....	....	......	3 889	5.07	7.0511	7.0056	554	554	4.46
−0.550	− 7.9864	−1.9575	+0.0239	− 9.9200	0.9900	2 335	6 603	....	......	......	....	......	......
−0.500	− 9.2594	−1.7796	+0.0198	−11.0182	1.0982	2 591	8 594	....	......	......	....	......	......
−0.487	....	....	....	....	....	......	9 260	5.23	7.1616	7.1063	756	1 310	10.55
−0.450	−10.6659	−1.6015	+0.0160	−12.2514	1.2332	2 909	11 503	....	......	......	....	......	......
−0.408	....	....	....	....	....	......	14 260	5.38	7.2636	7.2126	693	2 003	16.13
−0.400	−12.2394	−1.4236	+0.0126	−13.6504	1.3990	3 300	14 803	....	......	......	....	......	......
−0.350	−14.0225	−1.2456	+0.0097	−15.2584	1.6080	3 793	18 596	....	......	......	....	......	......
−0.340	....	....	....	....	....	......	19 446	5.49	7.3373	7.3004	710	2 713	22.26
−0.300	−16.0822	−1.0677	+0.0071	−17.1428	1.8844	4 445	23 041	....	......	......	....	......	......
−0.286	....	....	....	....	....	......	24 456	5.59	7.4040	7.3706	680	3 393	27.73
−0.270	−17.4755	−0.9609	+0.0058	−18.4406	1.2978	3 061	26 102	5.63	7.4303	7.4171	208	3 601	29.40
Second Component Wave, Positive.													
+0.270	−17.4755	+0.9609	+0.0058	−16.5088	....	......	26 000	....	......	......	....	3 601	29.40
+0.258	....	....	....	....	....	......	27 544	....	......	......	208	3 809	31.08
+0.250	−18.5175	+0.8897	+0.0049	−17.6129	1.1041	2 604	28 706	....	......	......	....	......	......
+0.225	....	....	....	....	....	......	32 554	....	......	......	680	4 489	36.55
+0.200	−21.4970	+0.7118	+0.0032	−20.7820	3.1691	7 475	36 181	....	......	......	....	......	......
+0.192	....	....	....	....	....	......	37 740	....	......	......	710	5 199	42.68
+0.166	....	....	....	....	....	......	42 740	....	......	......	693	5 892	48.26
+0.150	−25.3403	+0.5339	+0.0018	− 24.8046	4.0226	9 488	45 669	....	......	......	....	......	......
+0.140	....	....	....	....	....	......	48 111	....	......	......	756	6 648	54.34
+0.123	−27.9914	+0.4378	+0.0012	−27.5524	2.7478	6 481	52 150	....	......	......	554	7 202	58.80
Third Component Wave, Negative.													
−0.123	−27.9914	−0.4378	+0.0012	−28.4280	....	......	52 150	....	......	......	....	7 202	58.80
−0.120	−28.3217	−0.4271	+0.0011	−28.7477	0.3197	754	52 904	....	......	......	....	......	......
−0.109	....	....	....	....	....	......	55 889	....	......	......	554	7 756	63.26
−0.100	−30.7571	−0.3559	+0.0008	−31.1122	2.3645	5 577	58 481	....	......	......	....	......	......
−0.092	....	....	....	....	....	......	61 260	....	......	......	756	8 512	69.34
−0.085	−32.9270	−0.3025	+0.0006	−33.2289	2.1167	4 993	63 474	....	......	......	....	......	......
−0.078	....	....	....	....	....	......	66 260	....	......	......	693	9 205	74.92
−0.070	−35.5213	−0.2491	+0.0004	−35.7700	2.5411	5 994	69 468	....	......	......	....	......	......
−0.066	....	....	....	....	....	......	71 446	....	......	......	710	9 915	81.05
−0.060	−37.5805	−0.2135	+0.0003	−37.7937	2.0237	4 763	74 231	....	......	......	....	......	......
−0.056	....	....	....	....	....	......	76 456	....	......	......	680	10 595	86.52
−0.053	−39.2370	−0.1815	+0.0002	−39.4183	1.6246	3 815	78 046	....	......	......	208	10 803	88.20
Fourth Component Wave, Positive.‡													
+0.053	....	....	....	....	....	......	78 000	....	......	......	....	10 803	88.20
+0.050	....	....	....	....	....	......	79 544	....	......	......	208	11 011	89.88
+0.040	....	....	....	....	....	......	84 554	....	......	......	680	11 691	95.35
+0.030	....	....	....	....	....	......	89 740	....	......	......	710	12 401	101.48
+0.020	....	....	....	....	....	......	94 740	....	......	......	693	13 094	107.06
+0.010	....	....	....	....	....	......	100 111	....	......	......	756	13 850	113.14
+0.000	....	....	....	....	....	......	104 000	....	......	......	554	14 404	117.60

* Δx is the distance between stations corresponding to selected values of H.

† These figures are the means of those in the preceding column.

‡ As this wave is very small, it is assumed to become extinguished progressively in this reflection.

and for the negative wave (Table 2):

$$h = -0.595 \cos 2\pi \left(\frac{t - 554}{T}\right) + 0.14 \cos 2\pi \left(\frac{t - 6\,648}{T}\right)$$
$$- 0.109 \cos 2\pi \left(\frac{t - 7\,756}{T}\right) + 0.02 \cos 2\pi \left(\frac{t - 13\,850}{T}\right) \ldots\ldots (62)$$

At Station 9260, the equation for the positive wave is:

$$h = +0.537 \cos 2\pi \left(\frac{t - 1\,188}{T}\right) - 0.190 \cos 2\pi \left(\frac{t - 5\,606}{T}\right)$$
$$- 0.105 \cos 2\pi \left(\frac{t - 7\,982}{T}\right) - 0.02 \cos 2\pi \left(\frac{t - 12\,400}{T}\right) \ldots\ldots (63)$$

and for the negative wave:

$$h = -0.487 \cos 2\pi \left(\frac{t - 1\,310}{T}\right) + 0.166 \cos 2\pi \left(\frac{t - 5\,892}{T}\right)$$
$$- 0.092 \cos 2\pi \left(\frac{t - 8\,512}{T}\right) + 0.02 \cos 2\pi \left(\frac{t - 13\,094}{T}\right) \ldots\ldots (64)$$

These general equations may then be transformed—by the methods described for Equation (47)—into the simpler forms given below. Each one gives, directly, the maximum value of h, and the time required for the resultant wave to be propagated from the entrance to the point being considered, that is, the time of occurrence of high water.

The transformed equations are as follows: For Station 3889, the positive wave is,

$$h = +0.5864 \cos 2\pi \left(\frac{t - 190}{T}\right) \ldots\ldots\ldots\ldots (65)$$

and the negative wave is,

$$h = -0.5648 \cos 2\pi \left(\frac{t - 263}{T}\right) \ldots\ldots\ldots\ldots (66)$$

At Station 9260, the positive wave is:

$$h = +0.4456 \cos 2\pi \left(\frac{t - 483}{T}\right) \ldots\ldots\ldots\ldots (67)$$

the negative wave:

$$h = -0.4073 \cos 2\pi \left(\frac{t - 556}{T}\right) \ldots\ldots\ldots\ldots (68)$$

By giving proper values to t, instantaneous profiles may be constructed as shown in Fig. 10. Similar equations for velocity at the same stations may be found from the Equations (61) to (64) as explained for Equation (49), and from those equations instantaneous velocity curves may be plotted as shown in Fig. 11.

Comparing Figs. 9 and 10, it will be noted that the instantaneous profiles are very similar and that all the discrepancies are of a minor character.

Heights.—Bourdelles' curves are based on a complete symmetry between the positive and negative wave curves.* Under any known theory of wave propagation complete symmetry can exist only when the height of the wave is negligible with respect to the depth, which is not the case in the present instance. It is

* "Etude sur le Régime de la Marée dans le Canal de Suez," *Annales des Ponts et Chaussees*, 1898, 3me trimestre.

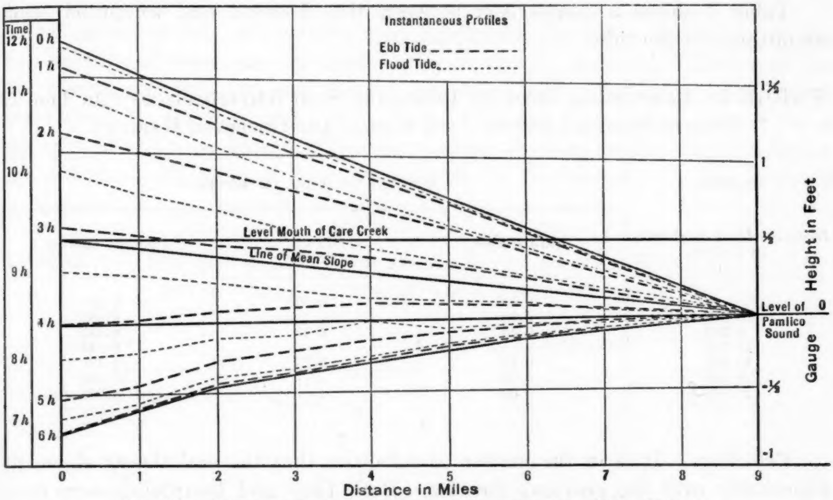


FIG. 10.—INSTANTANEOUS PROFILES COMPUTED FOR SUEZ CANAL.

the practice of the United States Coast and Geodetic Survey to allow for lack of symmetry between positive and negative waves in making tidal predictions by introducing a "shallow-water" component into the computations.* The new theory automatically allows for the shallow-water component when a separate computation is made for the negative wave.

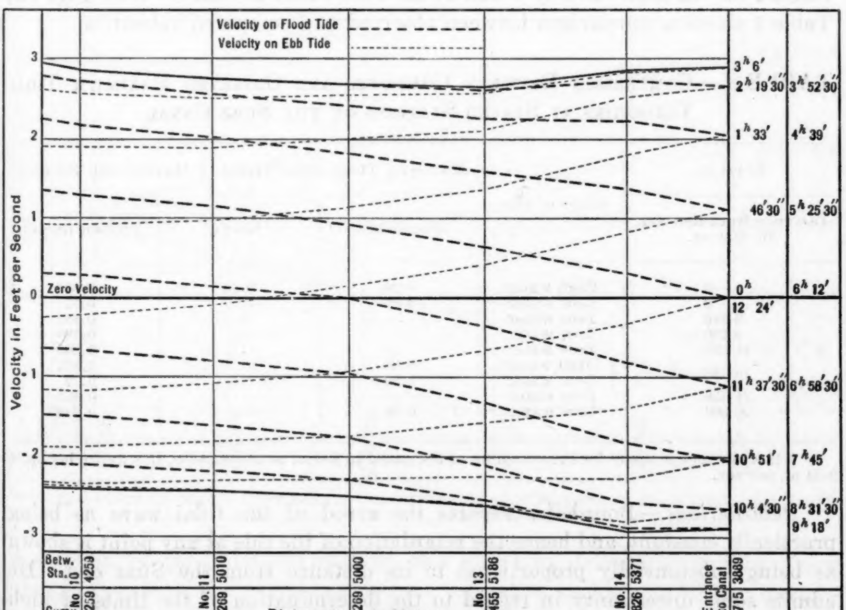


FIG. 11.—INSTANTANEOUS VELOCITY CURVES CORRESPONDING TO FIG. 10.

* *Special Publications, No. 111,* U. S. Coast and Geodetic Survey, p. 14.

Table 3 shows a comparison between the observed and computed semi-amplitudes of the tide.

TABLE 3.—COMPARISON BETWEEN COMPUTED SEMI-AMPLITUDES OF THE TIDE AT STATED STATIONS ON THE SUEZ CANAL, AND OBSERVED HEIGHTS.

STATION.	HEIGHTS OF TIDE, IN METERS.		
Distance from Red Sea, in meters.	Bourdelles.	Levy.	Present theory.
0	0.69	0.69	0.69
3 889	0.58	....	0.586
9 260	0.45	0.47	0.445
14 260	....	....	0.313
19 446	0.18	0.23	0.178
24 456	0.05	....	0.044
26 000	0.00	....	0.000

Velocities.—It is in the matter of velocities that the new theory shows its superiority over the previous theories. Both Lévy and Bourdelles were compelled to confess the failure of any theory of tidal propagation to explain the discrepancies noted between observed and computed velocities. These discrepancies were small at the Suez end of the canal, but they increased in magnitude until at the end at the Bitter Lakes there was complete disagreement. The velocities at all points in the canal, as computed by the new theory, are in satisfactory agreement with those observed, and it fully explains the reasons for the high velocity found at the outlet into the lakes. (See Fig. 11.) Table 4 shows a comparison between observed and computed velocities.

TABLE 4.—COMPARISON BETWEEN COMPUTED AND OBSERVED MAXIMUM TIDAL VELOCITIES AT STATED STATIONS ON THE SUEZ CANAL.

STATION.	Stage of tide.	MAXIMUM TIDAL VELOCITIES, IN METERS PER SECOND.		
Distance from Red Sea, in meters.		Bourdelles.	Levy.	Present theory.
0	High water	0.90	0.28	0.860
0	Low water	0.80	....	0.71
3 889	Low water	....	....	0.855
9 260	Low water	....	....	0.785
14 260	Low water	....	....	0.805
19 446	High water	0.85	....	0.824
19 446	Low water	0.80	....	0.72
24 456	Low water	....	....	0.858
26 000	Low water	0.50	....	0.880*

* If allowance is made for the widening of the canal to 500 m. at this station, this figure becomes 0.54 m. per sec.

Retardations.—Bourdelles reports the speed of the tidal wave as being practically constant, and hence the retardation of the tide at any point is shown as being substantially proportional to its distance from the Suez end. He admits some uncertainty in regard to the determination of the times of high and low water "by reason of the imperfections in the mode of observation used,

which consisted only in readings upon graduated scales".* The theory of the interference of waves as explained herein under the heading, "Theory of the Interference of Two Opposed Waves", requires that the speed of the resultant wave shall be variable, and not constant, although it will be constant for a distance of one wave length. The retardations as computed by the new theory are not markedly different from those given by Bourdelles; yet they conform to theoretical requirements (see Table 5).

TABLE 5.—THE RETARDATIONS OF HIGH WATER ON THE SUEZ CANAL; COMPUTED VALUES COMPARED WITH THOSE OBSERVED.

STATION.	RETARDATIONS, IN MINUTES.		
Distance from Red Sea, in meters.	Bourdelles.	Levy.	Present theory.
0	0	0	0
3 889	8	10	3
9 260	17	22	8
14 260	....	....	14
19 446	35	38	15
24 456	44	42	32
26 000	....	....	55

Inland Waterway from Pamlico Sound to Beaufort Inlet, N. C.—This is an artificial canal connecting the waters named, which was constructed by the United States about 1910. In 1911, some observations† were made of

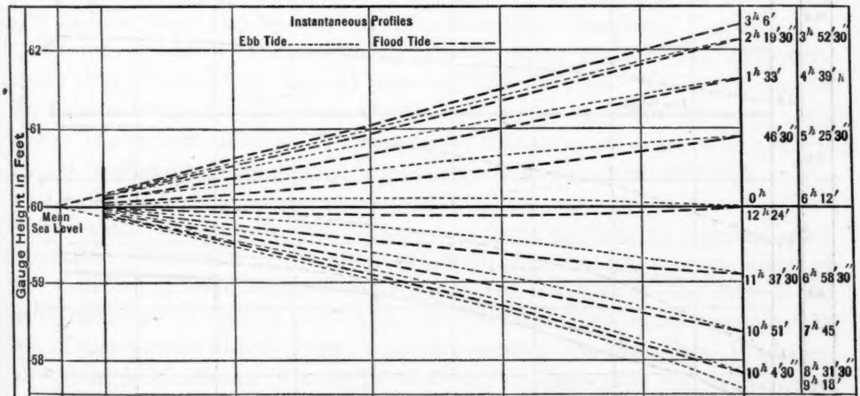


FIG. 12.—INSTANTANEOUS PROFILES AS COMPUTED FOR THE WATERWAY FROM BEAUFORT INLET TO PAMLICO SOUND, NORTH CAROLINA.

the action of the tidal currents in this canal to determine whether there were prospects of the currents attaining velocities such as to hinder navigation, and to require the installation of guard locks.

* "Etude sur le Régime de la Marée dans le Canal de Suez," par Bourdelles, *Annales des Ponts et Chaussees*, 3me trimestre, 1898.

† *Professional Memoirs*, Corps of Engrs., U. S. A., 1912, Vol. 4, p. 216.

Briefly, this waterway comprises three sections: The tidal stream known as Core Creek at the southern end; the connecting artificial canal; and the non-tidal Adams Creek at the northern end. The dimensions of the canal were 90 ft. wide on the bottom, with side slopes of 1 on 2, and 10 ft. deep referred to mean low water in Beaufort Harbor, near-by. The mean level of Adams Creek was 1.4 ft. referred to the same datum. The portion of the canal herein considered is about 8½ miles long.

The mean amplitude of the tide was found to be about 2.6 ft. at the mouth of Core Creek; thence it gradually decreased until all tidal variation of level practically disappeared at Hooker's Landing at the head of Adams Creek. Conversely, tidal velocities were mildest at the mouth of Core Creek, and greatest at Hooker's Landing. Some change of level was found at Hooker's Landing, but it was due entirely to wind effects on the level of Pamlico Sound and River into which Adams Creek empties. Curves of instantaneous heights and velocities for this canal have been computed under the proposed theory, as shown in Figs. 12 and 13, respectively. These may be compared with similar curves of observed data prepared by the Corps of Engineers, U. S. Army.

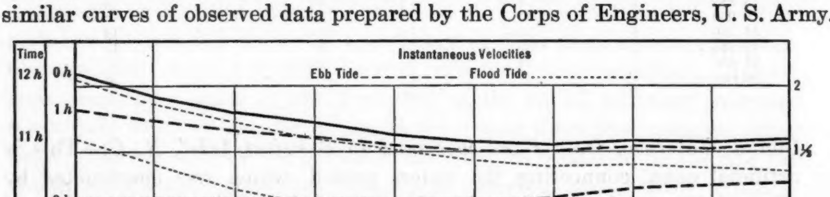


FIG. 13.—INSTANTANEOUS VELOCITIES AS COMPUTED FOR THE WATERWAY FROM BEAUFORT INLET TO PAMLICO SOUND, NORTH CAROLINA.

It will be noted that the mean level at the mouth of Core Creek is higher than that at Hooker's Landing, thus increasing the tidal velocities by a permanent set of the current northward due to this slope. This difference in level has been taken into account in computing the curves in Figs. 12 and 13.

The Rehoboth Canal.—It is now proposed to test the foregoing theory in a more complex example. The Inland Waterway between Rehoboth Bay and Delaware Bay, Delaware, is a tidal canal situated in the southeasterly portion of Sussex County, Delaware. It extends, for a distance of 12 miles, northward from Rehoboth Bay, through the highlands west of the Town of Rehoboth; thence through the marshes back of Cape Henlopen to Lewes River, a small tidal creek; and thence it follows this creek to Broadkill River at a point near the mouth of that stream, through which it empties into Delaware Bay about 5 miles from Cape Henlopen.

This canal was constructed by the United States about 1913, under a project which provides for a waterway 6 ft. deep at mean low water, 50 ft. wide through Lewes River and the marshes, and 40 ft. wide where it passes through deeper cutting.

Rehoboth Bay and Indian River Bay are two "drowned" valleys which formerly communicated with the Atlantic Ocean through an inlet. This inlet has closed and the two bays are completely separated from the ocean except by the Rehoboth Canal on the north and the Assowoman Canal on the south. The two bays, which are of about equal size, and which have a combined area of about 20 sq. miles, are partly separated from each other by a series of islands known as Burton's Islands, the connecting channels between the islands being locally known as the "Ditches". Those portions of the bays adjacent to the littoral cordon are quite shoal, but the average depth of the two bays elsewhere is about 6 ft.

The Assowoman Canal is a small artificial cut, originally intended to be 6 ft. deep and from 40 to 50 ft. wide. It connects Chincoteague Bay, Virginia, with Indian River Bay by way of Sinepuxent Bay, Maryland.

The surface level of Rehoboth and Indian River Bays is now quite constant; it is affected very little by anything but the wind. Tidal flow reaches the bays through the Rehoboth Canal, but has no apparent effect on their level. The current through the Assowoman Canal is not tidal, although the normal outflowing current may occasionally be reversed in direction by the wind.

The canal is naturally divided into two parts differing in physical characteristics. The northerly portion, 23 663 ft. long, has an average width of water surface at mean sea level of 100 ft., and an average sectional area of 900 sq. ft., making the reduced depth 9 ft. This portion of the canal comprises the bed of the former Lewes Creek, a stream heading up near Cape Henlopen. The canal bed narrows to a width of 80 ft. at Lewes, but maintains about the same depth. From the end of this section 3 000 ft. east of Lewes an artificial cut extends, for 33 404 ft., to the jetties in Rehoboth Bay terminating the canal. This cut has an average width on the surface of 78 ft. and an average cross-section at mean tide of 548 sq. ft., or a reduced depth of 7 ft.

In the artificial cut, the propagation is regular, and in accordance with theory. The height of the wave gradually decreases until it disappears in Rehoboth Bay, while the tidal velocities retain higher values than a direct proportion to the height of tide. The surface heights and velocities conform

to the requirements of the theory of a resultant wave caused by the superposition on a primary wave of a reflected negative wave from the bay.

In view of the difference in depth and area of the two sections of the canal as just mentioned, the tidal wave cannot be expected to be propagated through the entire length of the canal as simply as it would if both depth and cross-sections were constant for the entire distance. At the point of junction of the artificial cut with the old creek bed, a portion of the wave will continue on to the head of the old creek where a reflection of the wave back upon itself must be expected. This reflected wave, combining with the original wave, may be expected to cause higher levels and smaller velocities in that portion of the canal near Lewes than would have occurred normally. Furthermore, the constriction of the channel at Lewes must generate a reflected positive wave, and the division of the wave at the junction point must generate a reflected negative wave. These reflected waves may be expected to have a more pronounced effect near Lewes than near Broadkill, and to exert that effect at an earlier instant. The writer's observations amply confirm this expectation. Fig. 14 shows curves of instantaneous heights as given by hourly gauge readings made in August, 1925. These curves reveal clearly the large gauge heights and the advanced phases of the tide at Lewes resulting from this change of section.

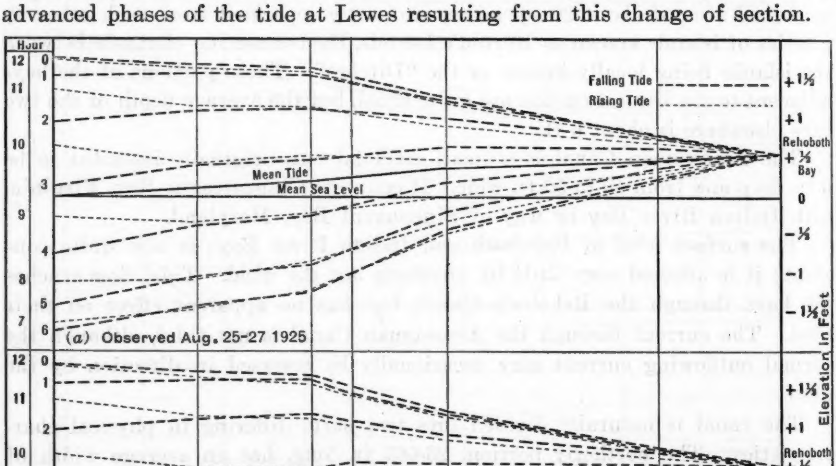


FIG. 14.—INSTANTANEOUS PROFILES IN THE LEWES-REHOBOTH CANAL, DELAWARE.

To find the resultant wave in this portion of the canal, first compute the direct wave, the positive reflected wave coming from the constriction at Lewes, the positive reflected wave from the head of the old creek, and the negative reflected wave from the junction with the artificial cut. A summation of these

waves will give the tidal characteristics of the Lewes River section. The remaining portion of the canal can be treated as being affected solely by the direct positive and the reflected negative wave.

Computation of Tidal Curves.—The tidal characteristics of the remainder of the canal may be determined by using the tidal curve at the junction as a generatrix of the tide between this point and the jetties.

Observations made in August, 1925, disclosed a maximum mean velocity of 1.41 ft. per sec. at the jetties, and a tidal amplitude of 2.8 ft. at the junction. Therefore, $H_1 = \frac{1}{2} \times 1.41 \sqrt{\frac{7}{32.17}} = 0.33$ ft., and the following values may then be determined for substitution in Equation (26): $H_0 = 1.4$ ft.; $x = 33\,404$ ft.; $D = 7$ ft.; $H_1 = 0.33$ ft.; $h = 78$ ft.; and $p = 83.4$ ft.

Making these substitutions, the constant, $\frac{F\,p}{2\,\rho\,l\,\sqrt{g}}$, is found to have the value, 0.00087877, from which, F is found to have the value, 0.5883. With this value of F in Equation (26), the height, H, at any point, x, along the canal may be found for both direct and reflected waves. Since the values of l and p are, respectively, 100 and 105 ft. for the Lewes River Section, the corresponding value of $\frac{F\,p}{2\,\rho\,l\,\sqrt{g}}$ will be 0.0008629.

The heights for each wave at selected points, as well as the times of propagation, may be computed as illustrated in Tables 1 and 2, and from these the phase angles are found. The general equations are then written for each point in a manner similar to the use made of Tables 1 and 2.

Fig. 13 is a graphical comparison between observed and computed instantaneous tidal curves.

Mean Level.—Attention has been called to the lack of symmetry between the geometric loci of high and low waters. If "mean canal level" is defined as a point midway between mean high water and mean low water at any point, it is quite evident that the locus of mean canal level so defined will not coincide with mean sea level which is the plane of reference. Tangent to mean sea level at the mouth, the locus of mean canal level will gradually ascend, and will be higher than mean sea level at the outlet in the reservoir, provided the reservoir can adapt itself to this condition. If not of unlimited capacity, it will so adapt itself in course of time.

If the reservoir surface should be below mean canal level at the outlet, the direct negative wave cannot return to the sea all the water brought in by the positive wave and an excess volume will remain in the reservoir; this excess must eventually bring it to the critical level. If the reservoir surface is above the mean canal level, the negative wave will return more water to the sea than was brought in by the positive wave, and eventually this must reduce the level to the proper elevation.

If the reservoir capacity is too great for such an adjustment to become effected, then the locus of mean canal level will incline downward from a point of maximum height and will become tangent to the reservoir level at the outlet.

It takes this form in any case during the periods of adjustment previously mentioned. Such a condition has been noted in connection with Adams Creek.

A fresh-water discharge into a comparatively small bay, such as those under consideration, will tend to raise the mean level above mean sea level.

The mean level of Rehoboth Bay is 0.55 ft. higher than mean sea level and the mean level of the canal rises gradually from one end to the other. This difference of level is to be attributed to the two causes just mentioned. Sufficient data do not exist upon which to determine the portions of this difference attributable to each cause. Measurements made at the jetties show a flood discharge of 11 070 090 cu. ft. per tide, and an ebb discharge of 12 230 330 cu. ft. per tide, a difference of 1 160 240 cu. ft. per tide, which represents a part of the fresh-water drainage into the basin. Proper corrections must be made to all computed values of tidal variation and tidal velocities in order to allow for the canal slope and fresh-water discharge. These corrections are made on the curves of instantaneous heights and velocities.

Comparison of Computed and Observed Data.—A series of tidal observations was taken upon the Rehoboth Canal at six selected stations during August, 1925, and gaugings were made at two stations near the respective ends of the canal. The height of the tide was read every 15 min. on a staff gauge at each station, the zero of which was established at the level of mean low water for Delaware Breakwater. These readings were used to determine mean sea level, mean range of tide, and mean high and mean low waters at each station.

Reduction of Observations.—The following method was used to check these determinations, and to give in addition the time of propagation of the tide from station to station.

If the time of making any reading upon a staff gauge is observed, and the height of the reading above mean sea level is known, the results of this observation may be used to write the equation,

$$h = H \sin 2\pi \left(\frac{t + c}{T}\right) \qquad (69)$$

in which, h and t are the observed height and time, respectively; H is the local half amplitude of the tide; and c is the time before the instant, $t = 0$, when the tide was at mean sea level; that is, when $t + c$ was equal to 0. It is clear that an independent equation of this form can be written for each observation, and, consequently, as many observation equations may be formed as are desirable for each tide, and they may be reduced by the methods of least squares to find the mean values of H and c.

If a series of observations are taken at several different stations, all reckoned from the same instant when $t = 0$, a value of c may be found for each point of observation, and the difference between the values of c for any two given points will be the time required for the tidal wave to be propagated between those stations.

In case the elevation of mean sea level is not known, the staff reading may be used directly, without a reduction, to determine the value of h.

Let R be the staff reading, the zero of the gauge being set at any desired elevation. Then, if e represents the elevation of mean sea level on the staff, $h = R - e$, and the observation equation may be written,

$$R = e + H \sin 2\pi \left(\frac{t + c}{T}\right) \ldots\ldots\ldots\ldots (70)$$

in which, e, H, and c are unknown.

Expanding,

$$R = e + \sin 2\pi \frac{t}{T} H \cos 2\pi \frac{c}{T} + \cos 2\pi \frac{t}{T} H \sin 2\pi \frac{c}{T} \ldots (71)$$

If n observations are made during any tide, n independent observation equations may be written, from which normal equations for each unknown quantity may be formed by the usual methods of least squares. These equations will be in the form,

$$\Sigma R = n\,e + H \cos 2\pi \frac{c}{T} \Sigma \sin 2\pi \frac{t}{T} + H \sin 2\pi \frac{c}{T} \Sigma \cos 2\pi \frac{t}{T} . (72)$$

$$\Sigma R \sin 2\pi \frac{t}{T} = e\,\Sigma \sin 2\pi \frac{t}{T} + H \cos 2\pi \frac{c}{T} \Sigma \sin^2 2\pi \frac{t}{T} + H \sin 2\pi \frac{c}{T} \Sigma \sin 2\pi \frac{t}{T} \cos 2\pi \frac{t}{T} \ldots\ldots (73)$$

$$\Sigma R \cos 2\pi \frac{t}{T} = e\,\Sigma \cos 2\pi \frac{t}{T} + H \cos 2\pi \frac{t}{T} \Sigma \sin 2\pi \frac{t}{T} \cos 2\pi \frac{t}{T} + H \sin 2\pi \frac{c}{T} \Sigma \cos^2 2\pi \frac{t}{T} \ldots\ldots (74)$$

In the observations at the Rehoboth Canal, hourly readings were used for this computation, and n was made 14 so as to embrace one complete tide in one set of equations. In this case, Equations (72), (73), and (74) become reduced to the following forms:

$$37.4 = 14\,e + 0.3267\,H \cos 2\pi \frac{c}{T} + 1.5376\,H \sin 2\pi \frac{c}{T} \ldots\ldots (75)$$

$$11.1686 = 0.3267\,e + 6.2924\,H \cos 2\pi \frac{c}{T} + 0.2135\,H \sin 2\pi \frac{c}{T} \ldots (76)$$

$$10.3442 = 1.5376\,e + 0.2135\,H \cos 2\pi \frac{c}{T} + 7.7076\,H \sin 2\pi \frac{c}{T} \ldots (77)$$

The left-hand members of Equations (75), (76), and (77) are the only ones that change for different stations, as t is counted for each station in all cases from the same instant. This makes the solution of a series of equations for observations at different points an easy process.

Table 6 gives the results of the solution of five sets of simultaneous observations at five points on the canal.

The values of c are given in Table 7, from which the time of propagation of the tide from the Broadkill Station to each of the others is obtained.

The differences are as listed in Table 8.

TABLE 6.—Computations for Simultaneous Observations at Five Points on the Canal.

(Observations are in feet.)

Item No.	Broadkill.		Greenhill.		Lewes.		Halfway.		Railway.	
	H.	*e.*	*H.*	*e.*	*H.*	*e.*	*H.*	*e.*	*H.*	*e.*
1....................	1.92	2.53	1.65	2.57	1.41	2.74	0.69	2.99	0.17	3.03
2....................	1.90	2.24	1.76	2.25	1.44	2.49	0.77	2.79	0.27	3.01
3....................	1.63	2.13	1.76	2.27	1.58	2.43	0.93	2.78	0.27	2.96
4....................	1.71	2.78	1.82	3.05	1.34	2.79	0.79	2.84	0.46	2.83
5....................	1.86	2.67	1.84	2.70	1.55	2.81	0.89	2.88	0.28	2.97
Mean..............	1.80	2.47	1.76	2.57	1.46	2.65	0.81	2.85	0.29	2.96
Computed..........	1.80	....	1.60	....	1.47	....	0.88	....	0.29	....

TABLE 7.—Values of *c* for Use in Determining Times of Propagation of the Tide.

Station.	Observation Set.				
	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.
Broadkill..............	29° 55′	60° 17′	255° 06′	40° 14′	53° 39′
Greenhill..............	16° 36′	52° 09′	252° 08′	31° 10′	50° 53′
Lewes..................	17° 57′	54° 00′	245° 09′	36° 44′	50° 46′
Halfway................	7° 48′	45° 04′	244° 10′	29° 26′	43° 41′
Railway................	0° 10′	51° 42′	203° 51′	10° 52′	46° 23′

TABLE 8.—Differences in Values of *c* as Referred to the Corresponding Value at Broadkill.

Set No.	From Broadkill to:			
	Greenhill.	Lewes.	Halfway.	Railway.
1..............................	13° 19′	11° 58′	22° 07′	30° 05′
2..............................	8° 08′	6° 17′	15° 13′	8° 35′
3..............................	2° 58′	9° 54′	10° 56′	51° 15′
4..............................	9° 04′	3° 30′	10° 48′	29° 12′
5..............................	2° 46′	2° 53′	9° 58′	7° 16′
Mean...........................	7° 25′	6° 54′	13° 48′	25° 17′
In seconds of time............	920	857	1 714	3 142
Computed values, in seconds..	574	1 248	1 450	1 900

It was noted during the period of observations that the canal surface was quite sensitive to the wind, and this fact is reflected in Table 8, but it is quite probable that that effect has been partly eliminated in the averages, and that they may be accepted as representative values. Since the computed times of propagation of the wave do not differ from the mean observed time by amounts greater than the mean error of the observed values, they may be accepted as confirmatory to the theory.

Canal Communicating with a Tidal Reservoir of Limited Capacity

It is now proposed to consider the case of the flow in a tidal canal having no proper tide of its own, but which connects a tidal sea with a basin of limited capacity such that, under the influence of the tide transmitted to it through the canal, its own surface undergoes periodic oscillations of level.

To explain the phenomena observed during the propagation of the tide in the type of canal now considered, assume as before that the wave propagated in the canal is assimilated to a series of small waves of translation. Consequently, it is formed during the flood by a series of successive intumescences or influxes of small height and by a similar series of depressions or refluxes during the ebb. These influxes and refluxes are propagated with the speeds and velocities determined by Equations (1) and (13), already given, conformably with the theory of waves of translation.

Assuming the sea, the canal, and the basin to be in equilibrium and at the same mean level, and that the first influx is then introduced, its action will be analogous to that of a canal communicating with a limitless reservoir except that the volume of the intumescence now has a finite relation to the volume of the basin, and will suffice, when introduced therein, to raise its level a certain amount. The rise of level will oppose the continued introduction of the intumescence into the basin, but this opposition will not be sufficient to counteract it completely until the volume of the intumescence poured into the basin shall equal in quantity the area of the basin multiplied by the height of the intumescence. When that occurs, the basin will be emitting into the canal in the direction of the sea, an opposed intumescence of equal height, the velocity due to which will completely annul that due to the first intumescence so that all flow will cease. Between the time of first arrival of the original intumescence at the outlet into the basin and the time when the basin intumescence exactly equals the original, the magnitude of the opposing basin intumescence will be increasing from zero to its final magnitude. Consequently, the effective magnitude of the original intumescence as an instrument for filling the basin to its own height will gradually diminish and will disappear when the basin is filled to that height.

In the case of an infinitely large reservoir as previously described, it was shown that as the intumescence was unable to raise the basin level, it was compelled to disappear in the basin through a lowering of its height and an increase of its velocity, resulting in a reflected depression of equal amplitude. Similarly, in the present instance, the intumescence must disappear in the basin, but the surface level of the basin will be rising, and, consequently, the reflected depression will not be equal to the original intumescence, but will be

equal only to the difference between the height of the intumescence and that of the basin surface at the same instant.

If the height of the intumescence as transmitted in the infinitely long canal and reduced by friction is h_l on its arrival at the basin, and if the height of the basin level at the same moment is h_b, the disappearance of the intumescence in the basin requires that any difference between h_l and h_b give rise to a reflected depression of height equal to $-(h_l - h_b)$, which is equivalent to considering a depression and an intumescence with respective heights equal to $-h_l$ and $+h_b$. The depression with a height equal to $-h_l$ is clearly the reflected depression corresponding to h_l already mentioned in the case of an infinitely large reservoir, and h_b is a new intumescence emitted by the basin, due to the rising of its level.

Similarly, a second intumescence propagating itself on the first one will raise the basin surface again to a point of equilibrium, thus giving rise to a similar series of phenomena, etc., for each influx.

As long as the successive influxes propagated into the canal and thence into the basin are greater in volume than the opposed fluxes emitted by the basin, the tide will be in flood, and the surface of the basin will continue to rise.

When the influxes shall have ceased to be further propagated, at high water, and the refluxes shall have begun to be propagated, these refluxes will add their velocity to that of the opposed fluxes from the basin. When the time arrives at which this sum is exactly equal to the combined velocity of the influxes, or when the total height of all intumescences is equal to the sum of the heights of all intumescences emitted from the basin, the basin level will cease to rise, and the velocity of inflow will be zero. Thenceforward, as the refluxes gain over the influxes, the surface of the basin will fall.

Manifestly, a similar reasoning on the refluxes of the negative wave would indicate an emptying of the basin until the influxes of a rising tide should cause a cessation of the emptying, and the resumption of a filling.

Previously, in discussing the properties of the wave of translation propagated in an infinitely long canal, the result of limiting the length by the introduction of an infinitely large reservoir was considered, and it was found that the result was to generate an opposed negative wave of magnitude equal to that of the original positive wave. The effect of limiting the length of such a canal by the introduction of a like reservoir, of limited capacity, will be similar, but with the added difference of causing the generation of a third wave—that emitted from the basin—which will be positive for a positive generatrix.

Therefore, the canal connecting the tidal sea with a basin of limited capacity will always have within it at least three simple waves of translation, and the actual heights and velocities at any point in it will be the resultant of those due to these three waves. The first will be the simple wave of translation generated by the tide in the sea, which will be propagated with proper friction losses as already explained for an infinitely long canal. The second wave will be the reflected negative wave corresponding to the first one exactly as previously explained. The third wave will be that emitted by the basin, positive for a positive tide and negative for a negative tide, and of a magnitude at the canal outlet in the basin that will be exactly equal to the height of the

basin level, but subject to the usual friction losses while being propagated in the canal itself.

Let the following additional notations be assumed:

H_b = the semi-amplitude of the basin tide.
t_b = the time of high water of the basin tide.
V_b = the velocity due to the height of wave, H_b.

The following equations may now be written for the tidal variation at the outlet into the basin,

$$h = H_1 \cos 2\pi\left(\frac{t - t_1}{T}\right) - H_2 \cos 2\pi\left(\frac{t - t_2}{T}\right) + H_b \cos 2\pi\left(\frac{t - t_b}{T}\right) \quad (78)$$

When $t = t_b$, then $h = H_b$; hence,

$$H_b = H_1 \cos 2\pi\left(\frac{t_b - t_1}{T}\right) - H_2 \cos 2\pi\left(\frac{t_b - t_2}{T}\right) + H_b \ldots\ldots (79)$$

Therefore,

$$H_1 \cos 2\pi\left(\frac{t_b - t_1}{T}\right) - H_2 \cos 2\pi\left(\frac{t_b - t_2}{T}\right) = 0 \ldots\ldots\ldots (80)$$

Since the direct wave and the reflected wave must have the same phase at the outlet into the basin, $t_1 = t_2$; hence, $H_1 = H_2$, and Equation (78) becomes

$$h = H_b \cos 2\pi\left(\frac{t - t_b}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (81)$$

The values of H_b and t_b will depend upon two factors, the first of which is the amplitude of the original wave of translation, and the second, the area of the basin. The first of these factors determines for a specific case the volume of the wave capable of being transmitted into the basin and available for filling it. The second determines the depth to which the filling may proceed, and, therefore, the proportion of the total volume of the wave of translation which is used for that purpose.

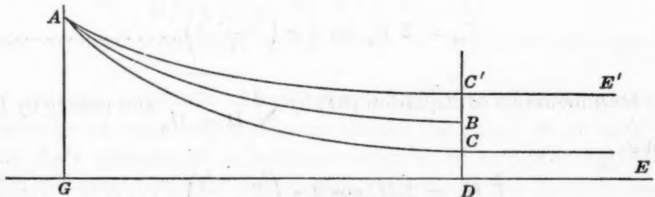


FIG. 15.

In Fig. 15, let $A\,B$ be the geometric locus of H as computed for the distance, $G\,D$, in an infinitely long canal having a semi-amplitude, $G\,A$, at G. The height, $D\,B$, will be the computed semi-amplitude of the wave, $A\,G$, as reduced by friction. Let $D\,B$ be the point at which the canal enters a basin the surface of which is represented by $C\,E$, and let $D\,C$ be the semi-amplitude of the surface variation of the level in the basin. Let it be assumed, also, for simplicity, that the area M, of the basin, is constant for all heights of the tide, so that its cubical contents above low-water level at any moment may be given by the expression, $M \times H_b\left(1 + \cos 2\pi\left(\frac{t - t_b}{T}\right)\right)$.

The heights, $D\,B$ and $D\,C$, will seldom be equal. Since the actual height of water in the canal will automatically adjust itself to that of the basin, the theoretical geometric locus of H will seldom coincide with the actual locus. If the basin rises only to $C\,E$, the geometric locus of high water in the canal will be $A\,C$ instead of $A\,B$. This means that the basin is of such large capacity that it receives not only the volume of the wave, $D\,B$, but also the volume equivalent to a wave the height of which is $C\,B$. If the basin is smaller, $D\,C$ may become larger than $B\,D$ as, for example, $D\,C'$. This condition would denote a basin with an area too small to afford a capacity to receive the entire volume of the wave, $D\,B$, and a surplus would be stored in the canal equal in volume to that of a wave with a height, $B\,C'$.

As shown in developing Equation (50), the velocity equation corresponding to Equation (78) may be written,

$$v = V_1 \cos 2\,\pi \left(\frac{t - t_1}{T}\right) + V_2 \cos 2\,\pi \left(\frac{t - t_2}{T}\right) - V_b \cos 2\,\pi \left(\frac{t - t_b}{T}\right) \text{.(82)}$$

and,

$$v = 2\,V_1 \cos 2\,\pi \left(\frac{t - t_1}{T}\right) - V_b \cos 2\,\pi \left(\frac{t - t_b}{T}\right) \ldots\ldots\ldots (83)$$

It has already been seen that when $t = t_b$, $v = 0$, hence,

$$0 = 2\,V_1 \cos 2\,\pi \left(\frac{t_b - t_1}{T}\right) - V_b$$

or,

$$V_b = 2\,V_1 \cos 2\,\pi \left(\frac{t_b - t_1}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (84)$$

Denoting the maximum velocity at the outlet by V_m, v will be equal to V_m when $t = t_b + \frac{T}{2}$. Hence,

$$V_m = 2\,V_1 \sin 2\,\pi \left(\frac{t_b - t_1}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (85)$$

Divide both members of Equation (84) by $\sqrt{\frac{g}{D + H_b}}$ and reduce by Equation (16), giving,

$$H_b = 2\,H_1 \cos 2\,\pi \left(\frac{t_b - t_1}{T}\right) \ldots\ldots\ldots\ldots\ldots\ldots (86)$$

Attention has already been called to the fact that the values of H_b and t_b depend on H_1 and M. Equation (86) shows the relation between these quantities and H_1.

An expression for H_b in terms of M is now required. By definition, the cross-sectional area through which the canal discharges into the reservoir at any instant, t, may be written, $a = A' + H_b\, l \cos 2\,\pi \left(\frac{t - t_b}{T}\right)$, A' being the area of the canal outlet at mean sea level, and equal to $D\,l$. The quantity of water discharged through this area in the time, dt, will be given by the equation, $dq = a\,v\,dt$.

Substituting in this equation the proper values of a and v expressed in terms of the variable, t, the equation for the differential discharge becomes,

$$dq = \left[D\,l + H_b\,l \cos 2\,\pi \left(\frac{t - t_b}{T}\right)\right] V_m \sin 2\,\pi \left(\frac{t - t_b}{T}\right) dt$$

$$= D\,l\,V_m \sin 2\,\pi \left(\frac{t - t_b}{T}\right) dt + H_b\,l\,V_m \cos 2\,\pi \left(\frac{t - t_b}{T}\right) \sin 2\,\pi \left(\frac{t - t_b}{T}\right) dt. \quad (87)$$

Whence, by integration,

$$Q = \int_{t_b}^{t_b + \frac{T}{2}} dq = \frac{D\,l\,V_m\,T}{\pi} \quad \ldots\ldots\ldots\ldots\ldots\ldots (88)$$

If Equations (84) and (85) are squared and added, $4\,V_1^2 = V_b^2 + V_m^2$. Whence,

$$V_m = \sqrt{4\,V_1^2 - V_b^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (89)$$

From Equation (16), $V_1 = \dfrac{\sqrt{g}\,H_1}{\sqrt{D + H_b}}$ and $V_b = \dfrac{\sqrt{g}\,H_b}{\sqrt{D + H_b}}$.

Substituting in Equation (89),

$$V_m = \sqrt{\frac{4\,g\,H_1^2}{D + H_b} - \frac{g\,H_b^2}{D + H_b}} = \sqrt{\frac{4\,g\,H_1^2 - g\,H_b^2}{D + H_b}} \quad \ldots\ldots (90)$$

giving for Q,

$$Q = \frac{D\,l\,T}{\pi} \sqrt{\frac{4\,g\,H_1^2 - g\,H_b^2}{D + H_b}} = 2\,M\,H_b \quad \ldots\ldots\ldots\ldots (91)$$

in which, H_b is the only unknown quantity.

Let it be assumed that $\dfrac{g\,D^2\,l^2\,T^2}{4\,M^2\,\pi^2} = K$, and reduce Equation (91) to the form, $H_b^2\,(D + H_b) = K\,(4\,H_1^2 - H_b^2)$, or,

$$H_b^3 + (D + K)\,H_b^2 = 4\,K\,H_1^2 \quad \ldots\ldots\ldots\ldots\ldots (92)$$

which is the required solution, K being the factor the value of which depends on M.

Should the semi-amplitude of any one or of all three of these principal component waves be of such magnitude, or should the canal be of such a short length that their subsequent reflections would be of sufficient importance to come within the usual limits of observation, they would require to be accounted for in the computation. For this purpose the corresponding semi-amplitudes of these various reflections will be designated H_3, H_4, etc., as before, for the various reflections of the sea wave, and H_{b3}, H_{b4} etc., for the semi-amplitudes of the reflections of the basin wave. The general equations for heights of tide and velocity of flow at any point in the canal will be, therefore,

$$h = H_1 \cos 2\,\pi \left(\frac{t - t_1}{T}\right) - H_2 \cos 2\,\pi \left(\frac{t - t_2}{T}\right) + H_3 \cos 2\,\pi \left(\frac{t - t_3}{T}\right)$$

$$- H_4 \cos 2\,\pi \left(\frac{t - t_4}{T}\right) + \text{etc.}, + \left[H_{b1} \cos 2\,\pi \left(\frac{t - t_{b1}}{T}\right)\right.$$

$$\left. - H_{b2} \cos 2\,\pi \left(\frac{t - t_{b2}}{T}\right) + H_{b3} \cos 2\,\pi \left(\frac{t - t_{b3}}{T}\right) - \text{etc.} \right] \quad \ldots\ldots (93)$$

$$v = V_1 \cos 2\pi\left(\frac{t - t_1}{T}\right) + V_2 \cos 2\pi\left(\frac{t - t_2}{T}\right) - V_3 \cos 2\pi\left(\frac{t - t_3}{T}\right)$$
$$+ V_4 \cos 2\pi\left(\frac{t - t_4}{T}\right) + \text{etc.}, - \left[V_{b1} \cos 2\pi\left(\frac{t - t_{b1}}{T}\right)\right.$$
$$\left. + V_{b2} \cos 2\pi\left(\frac{t - t_{b2}}{T}\right) - V_{b3} \cos 2\pi\left(\frac{t - t_{b3}}{T}\right) + \text{etc.}\right] \ldots\ldots(94)$$

The times of occurrence of the maximum heights and velocities at any point may be found by equations similar to Equations (53) and (54).

Inspection of Equations (84) and (86) shows that the maximum possible value of V_b is $2\,V_1$, and of H_b is $2\,H_1$, occurring when $t_b = t_1$. These substitutions in Equation (83) show the value of v to be zero for all values of t. This is manifestly the limiting case when the reservoir area has become so small as to disappear, and the canal has a "dead end".

The value of v is maximum at a time, $\frac{1}{4}\,T$, later than t_b. Its maximum possible value is $2\,V_1$, which can occur only when t_b and t_1 differ by $\frac{1}{4}\,T$; that is, when the basin is completely filled, or after the wave of translation has poured its entire volume into the basin. This can occur only when the basin is of such large extent that its level cannot be raised by this volume. It is apparent that if the level should be raised, an opposed wave would be generated, which would prevent the entire volume from entering the basin. This limiting condition then becomes the same as the case of an infinitely large reservoir.

It will probably be possible in any practical example of canal and basin of this type to observe both H_b and t_b directly, from which, H_1 can be found at once by means of Equation (86), and, consequently, the frictional coefficient by Equation (26). No example of this case in Nature is known to the writer with the exception of some small coastal bays connected with the sea by channels of various lengths and characteristics. Usually these channels are very short, since the bays are generally close to the ocean. The shorter the length of such channels, the greater will be the number of reflected waves that will be generated by the rise of the bay tide as well as by that of the sea tide. Equations (93) and (94) will then contain many terms in the last members, and their solution will be long and tedious. For such short canals the current may be assimilated, with possibly equal accuracy, to flow under the influence of a surface slope, always assuming the canal to be of short length. This method of solution has been described in a previous article.* It is the solution that will generally be found to be more suitable for this case, and of sufficient accuracy for all practical purposes.

Canal Connecting Two Bodies of Water Each Having an Independent Tide of Its Own

When a canal connects two bodies of water each having an independent tide of its own, no matter what the source of either tide may be, there occurs in the

* "Inlets on Sandy Coasts," by Earl I. Brown, *Proceedings*, Am. Soc. C. E., February, 1928, Papers and Discussions, pp. 505-553.

canal an interference of two opposed systems of waves, each system completely independent of the other. In the two cases heretofore discussed, all the various waves simultaneously affecting the surface of the canal could be considered as pertaining to a single system, since each and every wave was dependent in some manner on the primary wave of the sea as a generator. If the generating wave disappeared, all the various reflected waves also disappeared. The present case is that in which two such systems simultaneously affect the surface of the canal and are propagated in opposite directions.

Since the two systems are independent, it is apparent that the two primary generating waves—that is, the respective primary waves pertaining to the two bodies of water connected by the canal—may have any possible phase relation at a given point in it from complete coincidence to complete opposition, and their amplitudes may have any ratio between their respective magnitudes. If such amplitudes are equal and opposite in phase at certain points, there will be complete disappearance of variation in tidal height at these points. In general, the amplitudes at the two ends of the canal will not be equal, and the phase differences of the two generating waves will be greater than zero.

The method of solving the problem of determining the actual or resultant amplitude of the tides and velocity of flow at any given point in this case must be apparent from what has already been demonstrated in the two preceding cases.

If the subscripts, a and b, are used to characterize each wave system, respectively, the equation for height of the major and minor wave systems each will be,

$$h_a = H_{a1} \cos 2\pi \left(\frac{t - t_{a1}}{T}\right) - H_{a2} \cos 2\pi \left(\frac{t - t_{a2}}{T}\right) + H_{a3} \cos 2\pi \left(\frac{t - t_{a3}}{T}\right) - H_{a4} \cos 2\pi \left(\frac{t - t_{a4}}{T}\right) + \text{etc} \quad \ldots\ldots (95)$$

and,

$$h_b = H_{b1} \cos 2\pi \left(\frac{t - t_{b1} + d}{T}\right) - H_{b2} \cos 2\pi \left(\frac{t - t_{b2} + d}{T}\right) + H_{b3} \cos 2\pi \left(\frac{t - t_{b3} + d}{T}\right) - H_{b4} \cos 2\pi \left(\frac{t - t_{b4} + d}{T}\right) + \text{etc} \quad \ldots (96)$$

in which, d equals the difference in time between the arrival of high water at the respective ends of the canal. It is positive when the minor wave arrives the earlier.

The corresponding equations for velocity will be,

$$V_a = V_{a1} \cos 2\pi \left(\frac{t - t_{a1}}{T}\right) + V_{a2} \cos 2\pi \left(\frac{t - t_{a2}}{T}\right) - V_{a3} \cos 2\pi \left(\frac{t - t_{a3}}{T}\right) + \text{etc} \quad \ldots\ldots (97)$$

$$V_b = V_{b1} \cos 2\pi \left(\frac{t - t_{b1} + d}{T}\right) + V_{b2} \cos 2\pi \left(\frac{t - t_{b2} + d}{T}\right) - V_{b3} \cos 2\pi \left(\frac{t - t_{b3} + d}{T}\right) + \text{etc} \quad \ldots\ldots (98)$$

According to the principles discussed in connection with Equation (32), $h = h_a + h_b$ and $v = v_a - v_b$, or,

$$h = H_{a1} \cos 2\pi \left(\frac{t - t_{a1}}{T}\right) - H_{a2} \cos 2\pi \left(\frac{t - t_{a2}}{T}\right) + H_{a3} \cos 2\pi \left(\frac{t - t_{a3}}{T}\right)$$

$$- H_{a4} \cos 2\pi \left(\frac{t - t_{a4}}{T}\right) + H_{b1} \cos 2\pi \left(\frac{t - t_{b1} + d}{T}\right) - H_{b2} \cos 2\pi \left(\frac{t - t_{b2} + d}{T}\right)$$

$$+ H_{b3} \cos 2\pi \left(\frac{t - t_{b3} + d}{T}\right) - H_{b4} \cos 2\pi \left(\frac{t - t_{b4} + d}{T}\right) + \text{etc.} \quad (99)$$

and,

$$v = V_{a1} \cos 2\pi \left(\frac{t - t_{a1}}{T}\right) + V_{a2} \cos 2\pi \left(\frac{t - t_{a2}}{T}\right) - V_{a3} \cos 2\pi \left(\frac{t - t_{a3}}{T}\right)$$

$$- V_{b1} \cos 2\pi \left(\frac{t - t_{b1} + d}{T}\right) - V_{b2} \cos 2\pi \left(\frac{t - t_{b2} + d}{T}\right)$$

$$+ V_{b3} \cos 2\pi \left(\frac{t - t_{b3} + d}{T}\right) \quad (100)$$

To find the coefficient of friction: If Equation (100) is divided by $\sqrt{\dfrac{g}{D + h_a + h_b}}$ and then added to the actual head or height of tide (Equation (99)), thus,

$$h + v \sqrt{\frac{D + h_a + h_b}{g}} = 2 H_{a1} \cos 2\pi \left(\frac{t - t_{a1}}{T}\right) - 2 H_{b2} \cos 2\pi \left(\frac{t - t_{b2} + d}{T}\right)$$

$$+ 2 H_{b3} \cos 2\pi \left(\frac{t - t_{b3} + d}{T}\right) - 2 H_{b4} \cos 2\pi \left(\frac{t - t_{b4} + d}{T}\right) + \text{etc.} \quad (101)$$

It will be noted from Equation (96) that this expression may be written

$$h + v \sqrt{\frac{D + h_a + h_b}{g}} = h + h_v = 2 H_{a1} \cos 2\pi \left(\frac{t - t_{a1}}{T}\right)$$

$$+ 2 \left[h_b - H_{b1} \cos 2\pi \left(\frac{t - t_{b1} + d}{T}\right) \right] \quad (102)$$

whence,

$$h_a - h_b + h_v = 2 H_{a1} \cos 2\pi \left(\frac{t - t_{a1}}{T}\right) - 2 H_{b1} \cos 2\pi \left(\frac{t - t_{b1} + d}{T}\right). \quad (103)$$

At the end of the canal from which the major wave comes, $H_{a1} = H_{a0}$; $t_{a1} = 0$; $H_{b1} = H_{bL}$; $t_{b1} = t_{bL}$; $h_b = 0$; hence, $h_a + h_v = 2 H_{a0} \cos 2\pi \dfrac{t}{T} - 2 H_{bL} \cos \pi \left(\dfrac{t - t_{bL} + d}{T}\right)$. Substituting for h_a its equivalent, $H_{a0} \cos 2\pi \dfrac{t}{T}$, the value of h_v is found to be equivalent to,

$$h_v = H_{a0} \cos 2\pi \frac{t}{T} - 2 H_{bL} \cos 2\pi \left(\frac{t - t_{bL} + d}{T}\right) \quad (104)$$

For $t = 0$,

$$h_v = H_{a0} - 2 H_{bL} \cos 2\pi \left(\frac{t_{bL} + d}{T}\right) \quad (105)$$

At the end of the canal from which the minor wave comes, $H_{a1} = H_{aL}$; $t_{a1} = t_{aL}$; $H_{b1} = H_{b0}$; $t_{b1} = 0$; $h_a = 0$; whence,

$$- h_b + h_v = 2 H_{aL} \cos 2\pi \left(\frac{t - t_{aL}}{T}\right) - 2 H_{b0} \cos 2\pi \left(\frac{t + d}{T}\right). \quad (106)$$

Substituting, $h_b = H_{b0} \cos 2\pi \left(\frac{t + d}{T}\right)$, and $t = -d$, in Equation (106):

$$h_v = 2 H_{aL} \cos 2\pi \left(\frac{d + t_{aL}}{T}\right) - H_{b0} \ldots\ldots\ldots\ldots (107)$$

It is apparent from Equations (105) and (107) that the frictional coefficient can be computed from either of these combined with Equation (26), after measurement of the velocity at high water at either or both ends of the canal, and substituting the equivalent velocity head in the proper one of these equations, in which all terms will be known but H_{bL} or H_{aL}, and t_{bL} or t_{aL}. The latter may be computed with sufficiently close approximation from the known depths and heights of tide, thus fixing H_{bL} or H_{aL} for substitution in Equation (26). The measurement at the end from which the minor wave comes will usually be found the more satisfactory, thus giving H_{aL} which is larger than H_{bL}.

Application to Existing Canals.—There are a great many examples of canals of this type, such as the English Channel, the St. Georges Channel, Seymours Narrows, Deception Pass, the Cape Code Canal, and the Chesapeake and Delaware Canal.

The English Channel is an example of the longer canals of this type. Its tides are the resultant of the interference of two unequal opposed waves. The major or Atlantic wave enters from the south end, with an amplitude of 16 to 18 ft. The minor wave enters from the North Sea with an amplitude of about 10 ft.

The length of the Channel is great compared with that of the wave, hence there is sufficient room in it for the development of all the phenomena of wave interference anticipated from theoretical considerations. A complete description of these phenomena is given by Bourdelles* and will not be repeated here.

The case of the interference of two equal opposed waves is realized in the St. Georges Channel of the Irish Sea. Two flood currents having opposite directions, exist simultaneously and proceed from the ends of the channel toward the line of flood which forms a line joining the Isle of Man with Morecambe Bay, so that the sea rises on that line to a maximum height, but is accompanied by no current. Likewise, on the ebb, two opposed ebb currents start from this line directed toward the ends of the channel. At the place where the trough of one of the waves encounters the summit of the other, the vertical variation of the tide is zero, while the currents there reach their greatest velocity. This occurs at Coustown and at a point 50 miles south of Dublin. These ebb and flood currents turn throughout the channel at the time of high and low water at the line of flood; likewise, the time of maximum velocity throughout the channel corresponds to that of half tide at this line.

The Cape Cod Canal is an example of the shorter canals subject to unequal opposed waves.† The canal is about 8 miles long. The amplitude of the tide in Cape Cod Bay is about 9 ft. and of that in Buzzards Bay, 4 ft. This canal is so short with respect to the magnitude of its major wave that the number

* "Etude du Régime de la Marée dans la Manche," par Bourdelles, *Annales des Ponts et Chaussees*, 1899, 3me trimestre.

† "The Cape Cod Canal," by William Barclay Parsons, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1.

of reflections of the wave through the canal must be large, and the equations corresponding to Equations (99) and (100) would be long and tedious to solve. The writer has not attempted to compute by this theory the tidal characteristics of this canal because data obtained by observation known to him are not sufficiently complete for purposes of comparison.

The Chesapeake and Delaware Canal occupies an intermediate status as to length and depth between the Suez Canal and the Rehoboth Canal. It is the one by which the theory will be further tested. This canal connects the Delaware River with Chesapeake Bay.* It begins about 40 miles below Philadelphia, Pa., near Delaware City, Del., and extends for about 14 miles in a westerly direction to Chesapeake City, Md., on Back Creek, an arm of Chesapeake Bay. The canal was designed to afford a sea-level channel between these bodies of water, 12 ft. deep at mean low water, with a bottom width of 90 ft. The sea-level connection was made in February, 1927, and some observations were made on May 13, 1927, to ascertain the tidal conditions.

From previous records there was found to be an average higher high water of 6 ft. above mean low water at the Delaware entrance of the canal, and of 2.4 ft. at Chesapeake City. High water occurs two hours earlier at Chesapeake City than at the Delaware entrance.

The side slopes of the excavated prism are constructed at 2 ft. horizontal to 1 ft. vertical, and the bottom of the canal is uniformly 15 ft. below mean sea level, hence the following data result (see Fig. 16):

$$p = 33.5 + 33.5 + 90 = 157 \text{ ft.}$$
$$l = 30 + 30 + 90 = 150 \text{ ft.}$$
$$A = \frac{1}{2}(150 + 90) \times 15 = 1\,800 \text{ sq. ft.}$$
$$D = \frac{1\,800}{150} = 12 \text{ ft.}$$

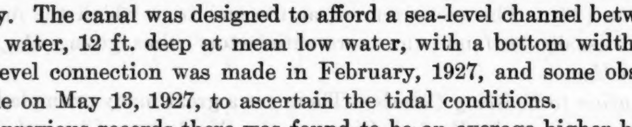


FIG. 16.—DIAGRAMMATIC CROSS-SECTION OF DELAWARE-CHESAPEAKE CANAL.

No opportunity was had to determine the exact value of the coefficient of friction for this canal. It has been shown, however, that a value of 0.51 was proper for channels of the cross-section and depth of the Suez Canal and of 0.58 for the Rehoboth Canal. The value of 0.55 appears to give results in accordance with observations for the Chesapeake and Delaware Canal, and that is the value adopted.

The first three miles of the canal at the Delaware end were constructed wider than the remainder, primarily to obtain more width for vessels to lie at the banks while awaiting passage, and, secondarily, to obtain material to build a dike in the vicinity. This change in width results in the heaping up of the tide at Mile 3, whch must be noted in considering the effect of width. It is

* *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 335.

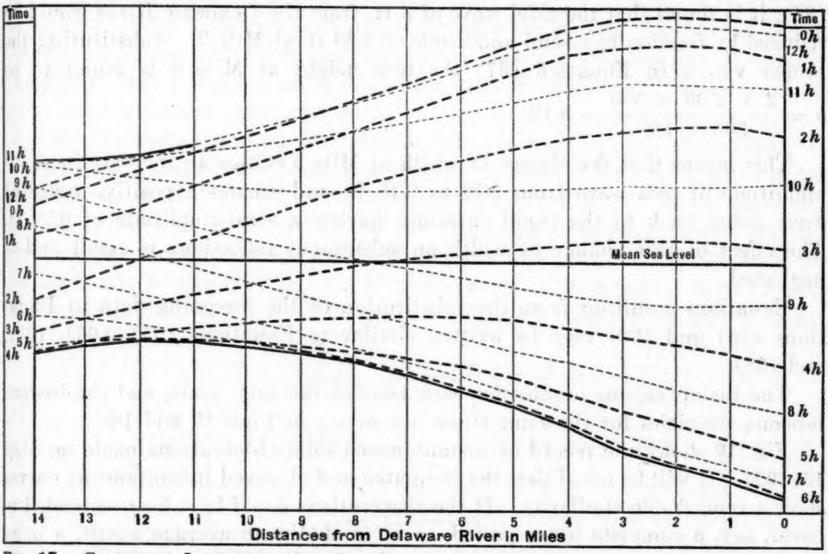


FIG. 17.—COMPUTED INSTANTANEOUS PROFILES IN THE CHESAPEAKE AND DELAWARE CANAL.

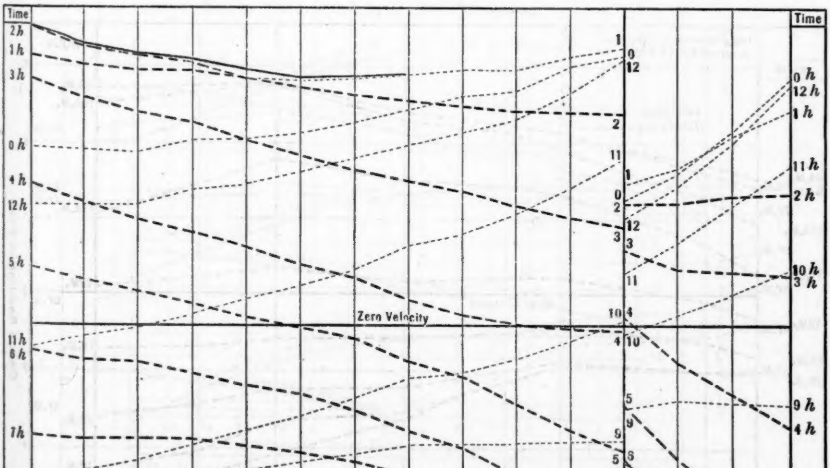


FIG. 18.—INSTANTANEOUS VELOCITIES AS COMPUTED FOR THE CHESAPEAKE AND DELAWARE CANAL.

300 ft. wide at the surface for these 3 miles. Using this width in Equation (26), it is found that the tidal wave of 3 ft. from the Delaware River would be reduced by friction to a semi-amplitude of 2.39 ft. at Mile 3. Substituting the proper values in Equation (37) the new height at Mile 3 is found to be $h = \dfrac{2 \times 2.39 \times 300}{300 + 150} = 3.18$.

This means that the change of width at Mile 3 causes an increase in semi-amplitude of this wave from 2.39 to 3.18 ft. and creates a positive reflected wave going back to the canal entrance, having a semi-amplitude of 0.79 ft. The effect of this change of width on subsequent reflections is small and is neglected.

Equations resulting from the substitution of the foregoing data in Equations (26) and (99) may be written similar to Equations (65), (66), (67), and (68).

The instantaneous profiles for each hour of the tidal cycle, and the instantaneous velocities for the same times are shown in Figs. 17 and 18.

Fig. 19 shows the record of instantaneous tidal observations made on May 13, 1927. It will be noted that the computed and observed instantaneous curves show a remarkable similarity. If the observations could have been extended to cover, say, a complete lunar period, so as to obtain an average result, a more complete comparison could have been made; yet the results are quite in agreement with the theory.

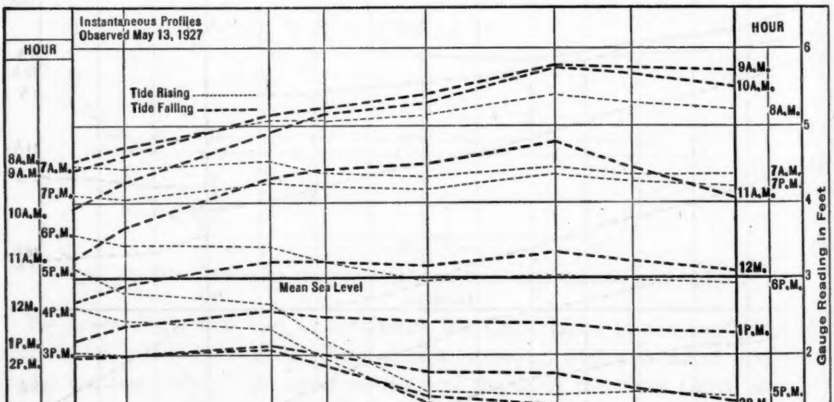
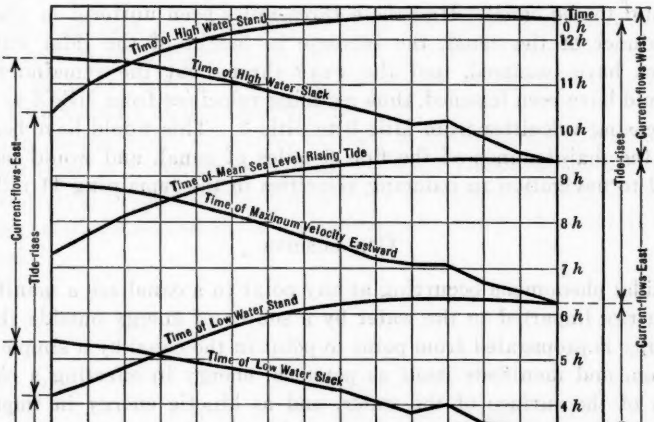


Fig. 19.—Observed Instantaneous Profiles in the Chesapeake and Delaware Canal.

It is regretted that the observations did not include current measurements, but the current meter broke down before the work was well begun. In a general way, however, velocities were found to conform to the indications of

theory. For instance, in the first three miles of the canal, the velocities are so low that silt brought in and deposited from the Delaware River which here has a very muddy bottom, has caused such rapid shoaling of the channel in this portion as to require much maintenance dredging. The sudden increase in velocity at Mile 3 is quite marked, and the greater velocity at Mile 14 was such as to cause shipping much concern.

Fig. 20 shows the computed retardations of high and low water, and the times of slack water and maximum velocities. These features were noted in the observations of May 13, 1927, and are also shown in Fig. 20. Here, again, the general appearance and trend of the curves is such as to show that the results given by theory agree quite well with observations. In conformity with the requirements of theory, the rate of propagation of high water is quite slow beginning at Mile 14, from which the rate gradually increases to Mile 6. From



NOTE:- The irregularities in curves at mile 3 are due to a change in width of Canal at that point

FIG. 20.—RETARDATIONS OF HIGH AND LOW WATER, AND OF MAXIMUM VELOCITIES, THE CHESAPEAKE AND DELAWARE CANAL.

the latter point to the canal entrance in Delaware River high water occurs practically simultaneously in the entire section. The point of minimum tidal variation is near Mile 13.

Beginning at high water at the Delaware entrance, the rate of propagation of the maximum velocity westward is fairly uniform in a westerly direc-

The only change of importance in this rate is at Mile 3, where irregularities are due to the change of width of the canal at that point. It will be noted that the time of maximum velocity westward occurs at Mile 14, somewhat after the level has fallen below mean sea level. Between Miles 11 and 12, slack water occurs on high and low-water stands, so that, in general, the period of falling tide corresponds to that of a westerly current and the period of rising tide to that of an easterly current. At the Delaware entrance, on the contrary, the current is westerly as long as the tide is above, and easterly as long as it is below, mean sea level. Mean canal level will be found above mean sea level at all points between its two ends.

Theory and observation both reveal a lack of similarity between the geometric loci of high and low water, due to the greater effect of friction on the negative waves. The effect of change of width at Mile 3 is more marked on the positive waves than on the negative. This change of width is shown to be very detrimental to the canal. Had these three miles been uniform in width with the remainder of the canal, the increase in height of the tidal wave there would not have occurred, and the wave throughout the remainder of the canal would have been lessened, thus reducing velocities from Mile 3 to Mile 14, and increasing velocities from Mile 0 to Mile 3. This would have been beneficial to the maintenance of the first 3 miles of canal, and would have been beneficial to navigation in reducing velocities in the remaining 11 miles.

Conclusions

The tidal phenomena occurring at any point in a canal are a manifestation of the energy imparted to the water by a source of energy outside the canal. This energy is propagated from point to point in the canal by a simple wave of translation, and manifests itself as potential energy in effecting a change of elevation of the surface of the water, and as kinetic energy in imparting a certain velocity of translation to the particles of water.

All the energy imparted to the water of a canal will ultimately become expended in overcoming the frictional resistance offered by the banks to the movement of the water.

If the canal is too short for this expenditure to become effected during the passage of a single simple wave of translation, the energy remaining unexpended when the simple wave has traversed the canal will be transmitted back through the canal and manifested in the form of a second simple wave of translation, termed a reflected wave.

If the canal terminates in a restricted or dead end, the reflected wave will be of the same sign as to amplitude as the direct wave, but of opposite sign as to velocity.

If the canal terminates in a widening or a basin of large capacity the reflected wave will be of opposite sign as to amplitude and of the same sign as to velocity, as the direct wave.

Whenever two or more simple waves of translation are simultaneously propagated in a canal, whether in the same or in opposite directions, a compound wave is formed, which is the resultant of all the simultaneous simple

waves. The resultant amplitude at any point is the algebraic sum of all the component amplitudes at that point, and the resultant velocity, the algebraic sum of all the component velocities, provided all velocities in the direction of propagation of the major wave are considered as positive, and all in the opposite direction, negative. In general, the algebraic sum of the component amplitudes will not bear the same simple relation to the algebraic sum of the component velocities as expressed in Equation (17) for the relation of amplitude to velocity in the simple wave of translation.

The simple wave of translation seldom exists in a state of Nature, and, consequently, can be but rarely observed. Practically all tidal propagation as observed in Nature is in the form of a resultant wave, in which the varying effect of the different components as modified by friction accounts for the varying relation so frequently observed between amplitude of tide and velocity of current at any given point.

Shallow channels are conducive to high velocities of tidal currents, with rapid absorption of energy, which reduces velocity. Deep channels are conducive to moderate current velocities, but absorb energy slowly, which tends to maintain currents at the same velocity.

As long as the width of a tidal channel is uniform, the magnitude of the width has little or no effect on velocity or other phases of tidal propagation. Any variation in the width of a tidal channel will affect the rate of expenditure of energy, and, consequently, it will affect both amplitudes and velocities in any canal.

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THE DON MARTIN PROJECT*

By Andrew Weiss,† M. Am. Soc. C. E.

Synopsis

On January 4, 1926, President Plutarco Elias Calles approved the existing Mexican Federal Irrigation Law. It provides in essence that the irrigation of private agricultural lands of whatever kind or extent of cultivation be declared a public interest, if subject to irrigation from Federal sources of water supply. Furthermore, it creates a National Commission of Irrigation composed of three members, to put into effect the studies of irrigation projects, the execution of all plans related thereto, and the administration of these within the provisions of this Act.

This paper gives a general description of the Don Martin Project, one of the largest so far undertaken by the Mexican Government under the provisions of this Act, including therein also a brief outline of the construction methods as well as of the settlement plans in mind and developed to date.

General Features of the Project

Location.—The Don Martin Project is on the Salado River in the States of Coahuila and Nuevo León. The Town of Rodriguez, about 45 miles southwest of Laredo, Tex. (see Fig. 1), is near the center of its irrigable area which embraces a total of 65 000 hectares (160 000 acres).

Irrigation Plan.—The storage feature of this system consists of a reservoir 100 km. (60 miles) directly west of Laredo; its normal capacity is 138 600 hectare-m. (1 123 600 acre-ft.) covering an area of 19 600 hectares (48 412 acres) when filled.

The dam affecting this storage is 66 km. (41 miles) up stream from Rodriguez and an equal distance west of Camaron at a place where a low range of hills known as "Lomerias de Caracol" crosses the Salado River in a general north-northeast and south-southwest direction. This dam will also

Note.—Written discussion on this paper will be closed in April, 1931, *Proceedings*.

* Presented before the Texas Section of the Society at the Annual Meeting, Laredo, Tex., November 14, 1928.

† Res. Engr., The J. G. White Eng. Corp., S. en C., Estacion La Cruz, Chihuahua, Mexico.

serve to divert the water impounded by it into the main canal of the system. After leaving the head-works this canal diverges slightly northward from the river for a distance of 42 km. (26 miles) from which point one of its principal branches, the Camaron Lateral, extends in a general easterly direction to cover the first unit to be irrigated in the vicinity of Camaron and Rodriguez. From this point also the main canal turns almost directly south a distance of about 120 km. (75 miles), and about 5 km. (3 miles) up stream from the crossing of the Laredo-Monterrey Highway with the Salado River. (See Fig. 2.) The area to be irrigated covers 30 000 hectares north of the river and 35 000 hectares south of it, or a total of 65 000 hectares (160 000 acres).

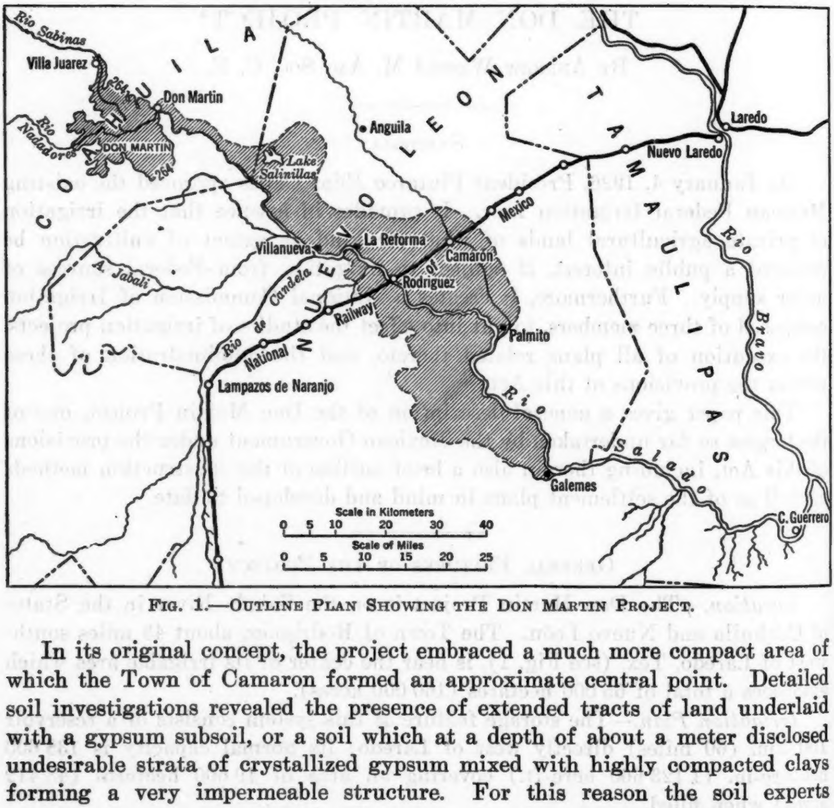

FIG. 1.—OUTLINE PLAN SHOWING THE DON MARTIN PROJECT.

In its original concept, the project embraced a much more compact area of which the Town of Camaron formed an approximate central point. Detailed soil investigations revealed the presence of extended tracts of land underlaid with a gypsum subsoil, or a soil which at a depth of about a meter disclosed undesirable strata of crystallized gypsum mixed with highly compacted clays forming a very impermeable structure. For this reason the soil experts selected the younger alluvial soils nearer the river and generally free from these gypsum bottoms. This choice of the major body of the project lands to the south of the river, although resulting in an appreciably higher estimated cost, seems to have been a wise move.

Water Supply.—The water supply for this project is almost wholly dependent upon the storage of flood waters of the Salado River and its upper tributaries, principal of which are the Sabinas and the Nadadores, which unite to form the Salado about 8 km. (5 miles) above the dam. The catchment basin

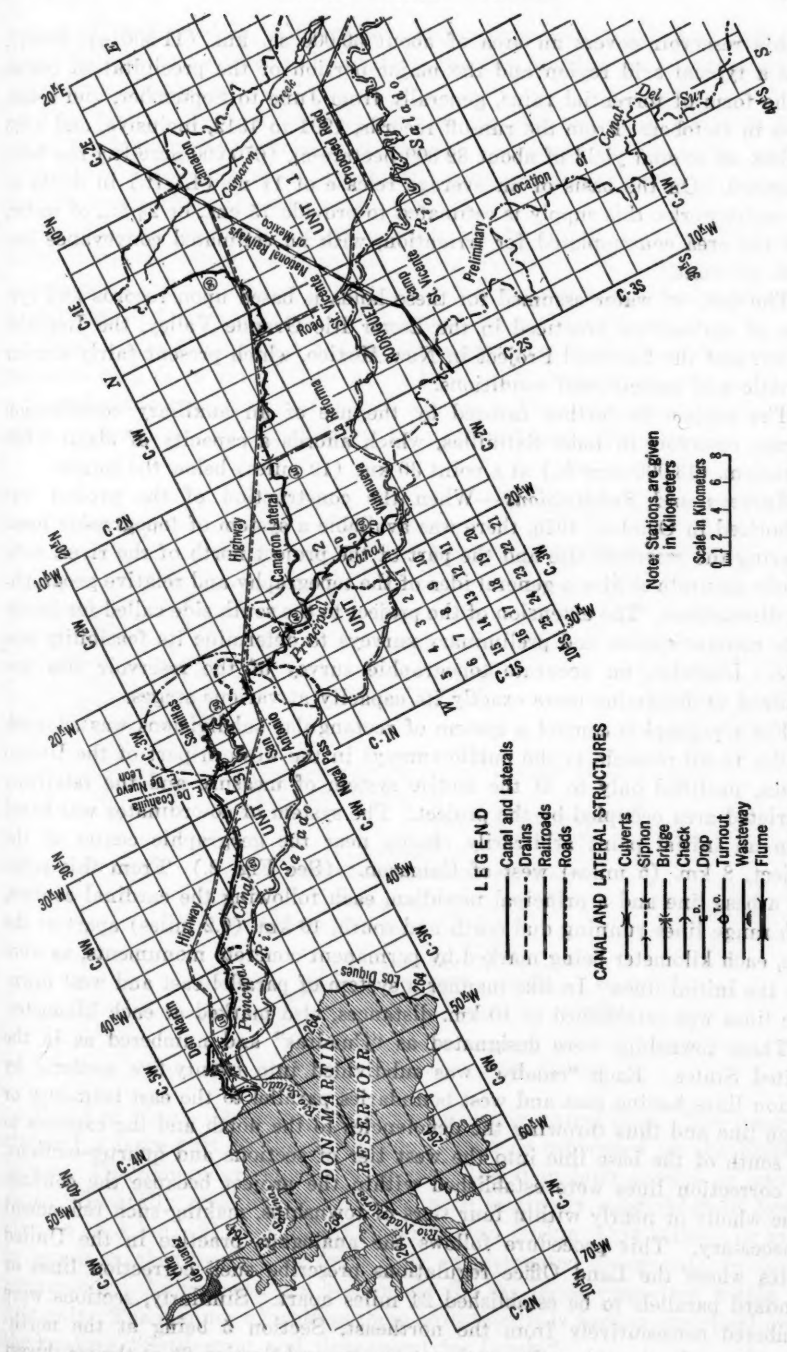

FIG. 2.—DETAIL PLAN SHOWING DON MARTIN PROJECT.

of this reservoir covers an area of about 30 000 sq. km. (11 500 sq. miles). It is a typical arid region and the major portion of the precipitation comes in the form of torrential rains, generally from June to September, and sometimes in October. From the run-off records, 1901 to 1912, inclusive, and 1923 to date, an annual yield of about 82 000 hectare-m. (670 000 acre-ft.) has been computed. On the basis of an average release of 1¼ m. (4.1 ft.) in depth at the outlet works, this supply is estimated to provide 75 cm., or 2½ ft., of water, over the area contemplated for irrigation, with an estimated conveyance loss of 40 per cent.

The duty of water assumed for these lands is based upon records and systems of agriculture practiced in the Lower Rio Grande Valley, the Imperial Valley, and the Carlsbad Project in New Mexico, which present fairly similar climatic and agricultural conditions.

The project is further favored by the use of an auxiliary equalization storage reservoir in Lake Salinillas, which affords a capacity of about 1 600 hectare-m. (13 000 acre-ft.) at a point 30 km. (19 miles) below the intake.

Surveys and Subdivisions.—When the construction of the project was authorized in October, 1926, there was available a system of topographic maps covering the reservoir site and the part of the project north of the river, sufficiently accurate to give a general idea of the topography and relative geographical dimensions. The extension of the project to the south side called for immediate reconnoissance and preliminary surveys to determine its feasibility and costs. Likewise, an accurate topographic survey of the reservoir site was required to determine more exactly its capacity at various stages.

For topographic control a system of rectangular subdivisions was adopted, similar in all respects to the public surveys in the western part of the United States, modified only to fit the metric system of measure and the relatively restricted area occupied by the project. The system of co-ordinates was based upon an initial point arbitrarily chosen near the geographic center of the project, 8 km. (5 miles) west of Camaron. (See Fig. 2.) From this point ran a base line and a principal meridian, each following the cardinal courses, with range lines running due north and south, 10 km. (6.2 miles) apart at the base, each kilometer being marked by permanent concrete monuments, as were also the initial lines. In like manner a system of parallel east and west township lines was established at 10-km. distances, also marked at each kilometer.

These townships were designated as "Cuadras" and numbered as in the United States. Each "cuadra" was subdivided into twenty-five sections, by section lines having east and west boundaries parallel to the east boundary or range line and thus throwing the deficiencies to the north and the excesses to the south of the base line into the west tier of sections and quarter-sections. No correction lines were established within the project because the confines came wholly or nearly within four tiers of townships, making such refinement unnecessary. This procedure follows the analogous practice in the United States where the Land Office regulations prescribe such correction lines or standard parallels to be established 24 miles apart. Similarly, sections were numbered consecutively from the northeast, Section 5 being at the northwest corner, Section 21 at the southeast corner, and Section 25 at the southwest

corner of each cuadra. All interior section lines were marked at each kilometer both as to location and elevation, thereby forming an excellent control for the topography.

This system of subdivision forms also the skeleton upon which is being based the farm unit subdivision, the location of minor distributaries, and the network of secondary roads which will give an outlet to the principal, or market, roads where the topography permits.

In addition, the location of farm units will be greatly facilitated by alphabetical lettering within each section. Thus, Farm Unit A (Cuadra * * *) is in each case the northeast parcel or subdivision within the section, thus distinguishing it from any other parcel within the entire project.

Roads.—The Commission has planned a system of public roads to give easy access to each farm unit. These are to be of such ample width that ultimately a line of shade trees may border each side, to enhance the beauty of the landscape, as was done on the Salt River Project, in Arizona, for example.

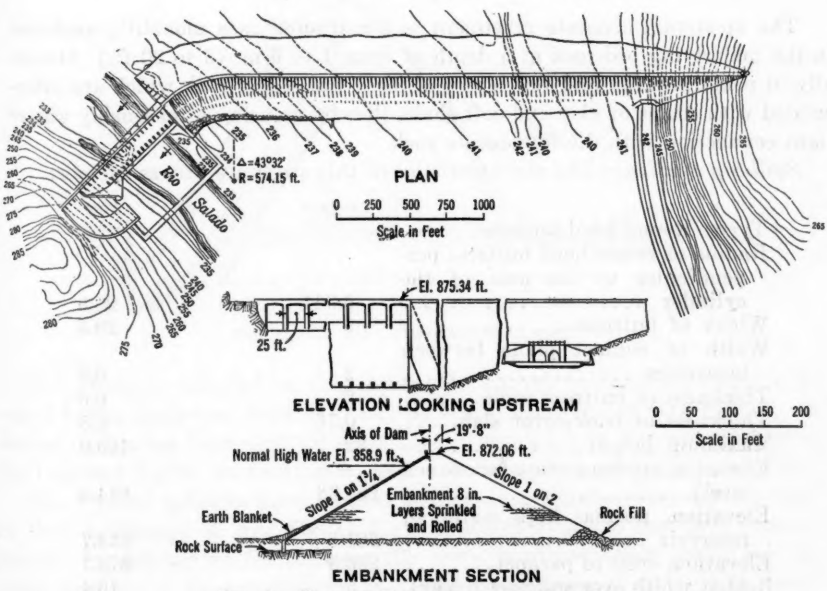

FIG. 3.—PLAN OF DAM ACROSS RIO SALADO.

Right of Way.—The entire project is privately owned land. The reservoir site and a large part of the north side is held or controlled by the Bank of Milmo. The remainder is owned by perhaps less than forty persons or associations. Generally speaking, these lands were originally subdivided in lots of about 4 to 8 km. (2½ to 5 miles) in width and 30 km. (18¾ miles), or more, in length, marked by existing fence lines supplemented occasionally by corner monuments. Adjustment has been made for the lands comprised within the first unit. The manner of making these adjustments is further explained under the heading, "Settlement and Repayment for Lands."

General Description of the Works

Rio Salado Dam—Earth and Gravel Fill.—This dam is a composite structure consisting of an earth and gravel embankment 985 m. (3 231 ft.) in length and a concrete overflow section 234 m. (768 ft.) in length. (See Fig. 3.) Following are the characteristics of the embankment:

	Meters.	Feet.
Length	985	3231
Greatest height	34.8	114
Up-stream slope	$1\frac{3}{4}$:1	$1\frac{3}{4}$:1
Down-stream slope	2:1	2:1
Width of crown	6	19.7
Free-board to crown of embankment	4	13.1
Free-board to top of parapet wall	5	16.4
Revetment: Reinforced concrete slab varying in thickness from 11 in. at the base to 8 in. at the crown, with $\frac{3}{4}$-in. round bars, 12 in. center to center, horizontally and vertically.		

The up-stream concrete revetment is constructed as a monolith, anchored to the underlying bed-rock at a depth of from 1 to 6 m. (3 to 20 ft.) Generally, it penetrates the first few layers of loose and seamy rock which are intercalated with seams of clay and soft shale, thereby assuring a reasonably watertight connection with the foundation rock.

Spillway Section.—The characteristics of this structure are, as follows:

	Meters.	Feet.
Type: Round-head buttress.		
Radius of round-head buttress perpendicular to the axis of the cylinder	6.364	20.9
Width of buttress	9	29.5
Width of contact plane between buttresses	2	6.6
Thickness of buttress walls	2	6.6
Thickness of back-cover slab	0.75	2.5
Maximum height	32	105.0
Elevation, spillway crest (above sea level)	257.38	844.2
Elevation, normal high water in reservoir	261.8	858.7
Elevation, crest of parapet	266.8	875.1
Bridge, width over spillway (clear)	5	16.4
Height of radial gates	4.42	14.5
Width of radial gates	7.62	25.0
Number of radial gates	26	26
Computed discharge capacity with surface elevation of water at 263.8 m. (865.3 ft.)	6 000 cu. m. per sec.	210 000 sec-ft.

The spillway is controlled by a system of twenty-two automatic radial gates, 25 by 14.5 ft., supplemented with four additional gates operated by direct mechanical lift. The water admitted through the 6-in. entrance at the bottom of the float-well exerts a buoyant force on the suspended float sufficient to bring about an overbalancing of the gate frame by action of the suspended

counterweight attached to the lever arm which projects 16 ft. beyond the gate axis, as shown in Fig. 4. The admission of the water which effects the filling of the float-well is automatic when the level of the reservoir rises above an elevation of 261.8 m. (858.7 ft.), and, upon cessation of inflow, the float-wells are automatically drained by a 3-in. pipe or orifice at the bottom. The admission of the water may also be regulated at will by a small shutter at the entrance of the 6-in. pipe at spillway level, if for any reason it should be desired to lower any or all of the gates before the water has receded to the normal high-water level, 261.8 m. (858.9 ft.).

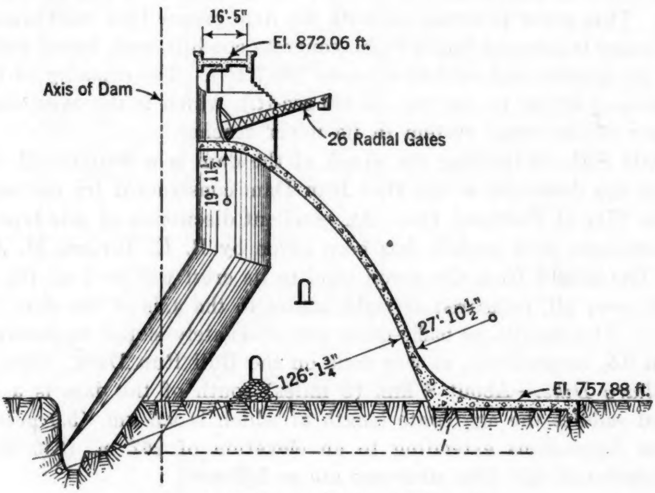

FIG. 4.—SECTION THROUGH SPILLWAY.

The principle adopted in this automatic control was independently conceived by the designing force of the J. G. White Engineering Corporation. A similar device has been used for some time in the Turlock Irrigation District, the Imperial Valley, in California, and elsewhere, with complete satisfaction.

The type of spillway here adopted follows in general a suggestion given* by Fred A. Noetzli, M. Am. Soc. C. E., with modifications to suit local requirements. Its distinctive features are the round-head buttresses conveying the water pressure to the counterforts by radial compression, thus avoiding bending moments and flexural stresses to be resisted by reinforcement. These were considered objectionable because of the highly corrosive action on steel of the waters in this stream. Furthermore, this type of independent units is peculiarly adapted to the local foundation conditions, each of the counterforts being deeply embedded into the limestone strata by a system of channeling, thus affording a maximum attainable sliding resistance within acceptable limits of cost.

Sluice-Gates.—Provision is also made for the complete drainage of the reservoir when desired through four 3.2 by 4-ft. rectangular sluice-gates,

* "The Design and Construction of Dams," by Edward Wegmann, M. Am. Soc. C. E., pp. 524-527.

operated by geared, hand-lifting devices of suitable design in the interior of the dam at an elevation of 241.26 m. above the floor of the down-stream apron. The gate-sill is at an elevation of 232.1 m., which is somewhat below the ordinary level of the water surface in the river at this point. This gate is shown in Fig. 5.

Outlet Works.—Water may be released from the reservoir through two batteries of eight vertical, rectangular, 3.5 by 10-ft. sliding gates, each battery containing four regulating and four emergency gates. (See Fig. 6.) The gate stems extend vertically to mechanical lifting devices operated by electric motors located in a gate-house at the top of the gate tower, at an elevation of 265.8 m. This tower is connected with the dam proper by a steel truss bridge.

The water is released into a 2-bbl. concrete conduit, each barrel built in the form of an elliptic arch of 5.02-m. span (16.7 ft.). The capacity of the gates is in excess of 65 cu. m. per sec. (2 296 sec-ft.), which is the calculated maximum duty of the canal system in its upper reaches.

Dentate Sill.—Adjoining the apron of the dam is a dentate sill of a type similar to the deflectors at the Bull Run Dam constructed for the water supply of the City of Portland, Ore. An excellent discussion of this type, including experiments with models, has been given by B. E. Torpen, M. Am. Soc. C. E.* The height from the apron level to its crest will be 2 m. (6½ ft.) and its length over all, measured at right angles to the axis of the dam, 13.22 m. (43.4 ft.). The dentils, or baffle-piers, are alternately sloped up stream 1 on 5 and 1 on 2.5, respectively, as was done on the Bull Run Dam. (See Fig. 7.)

Auxiliary Dike.—About 8 km. (5 miles) south of the dam is a series of connected saddles the aggregate length of which is 9.5 km. (5.9 miles), with minimum depressions extending to an elevation of 257 m. (843 ft.). The characteristics of this dike structure are as follows:

	Meters.	Feet.
Maximum height	8.8	28.9
Up-stream slope	1¾:1	1¾:1
Down-stream slope	2:1	2:1
Width of crown	6.	19.7
Revetment: Mamposteria stone masonry of lime mortar 0.26 m. (10 in.) in thickness, connected at its footing with an up-stream cut-off wall extending to a variable depth, generally 0.8 H (H representing the height of the dike, in meters).		
Height, parapet wall above crown	1.	3.28
Free-board to top of parapet wall	5	16
Quantity of earth-fill	651 540 cu. m.	852 195 cu. yd.
Quantity of intermediate cut-off trench excavation	38 720 cu. m.	50 645 cu. yd.

The body of this dike is composed of the earth material from shallow borrow-pits near-by and down stream. Because the ground here is underlaid at shallow depths with a gypsum stratum of undesirable composition for this purpose, it has been necessary to limit the depth of the borrow-pits.

* *Transactions*, Am. Soc. C. E., Vol. 93 (1929), pp. 487-496.

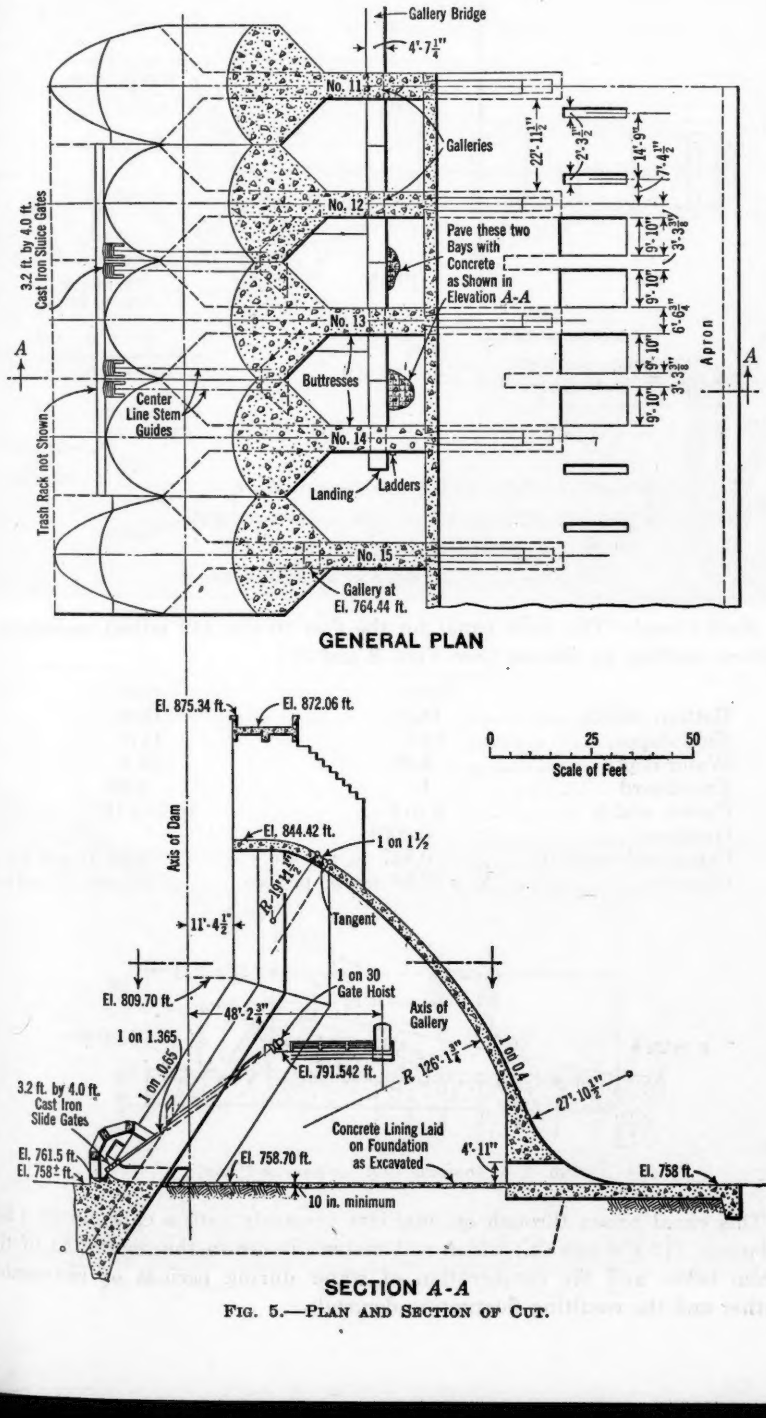

FIG. 5.—PLAN AND SECTION OF CUT.

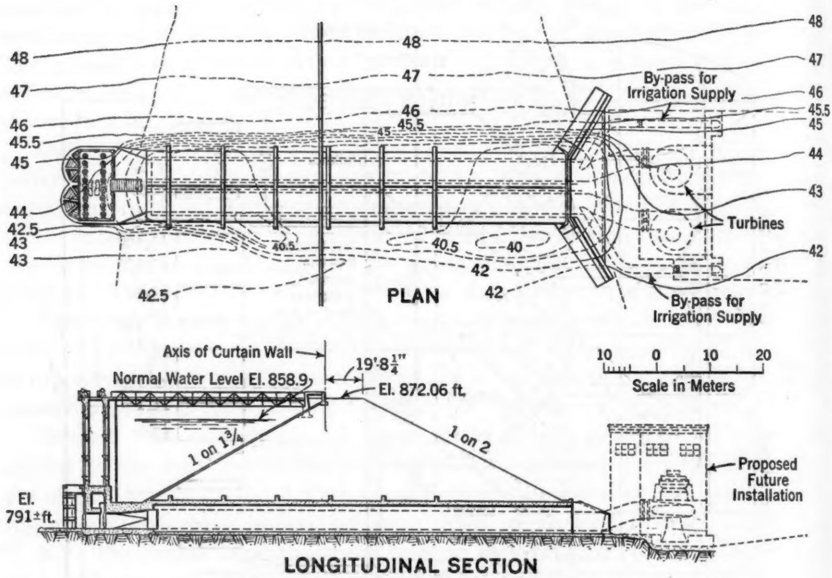

FIG. 6.—PLAN AND SECTION OF SLIDING GATES.

Main Canal.—The main canal for the first 30 km. (19 miles) maintains a uniform section, as follows (See Figs. 8 and 9):

	Meters.	Feet.
Bottom width...........	18.3	60.0
Side slopes.............	1½:1	1½:1
Water depth	3.23	10.6
Free-board	1.	3.28
Crown width	5 to 6	16.7 to 19.7
Gradient.................	0.0001	
Computed velocity.......	0.847 m. per sec.	2.78 ft. per sec.
Capacity	63.36 cu. m. per sec.	2 239 cu. ft. per sec.

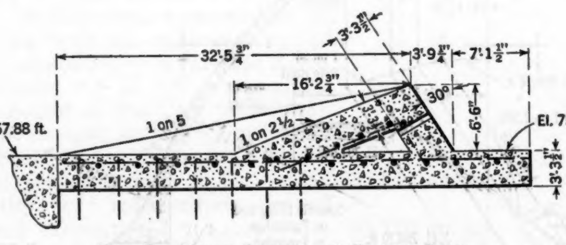

FIG. 7.—CROSS-SECTION OF BAFFLE-PIERS.

This canal passes through an auxiliary reservoir with a capacity of 1 600 hectare-m. (12 970 acre-ft.) which will materially aid in the regulation of the system below and the conservation of water during periods of changeable weather and the resulting fluctuating demands.

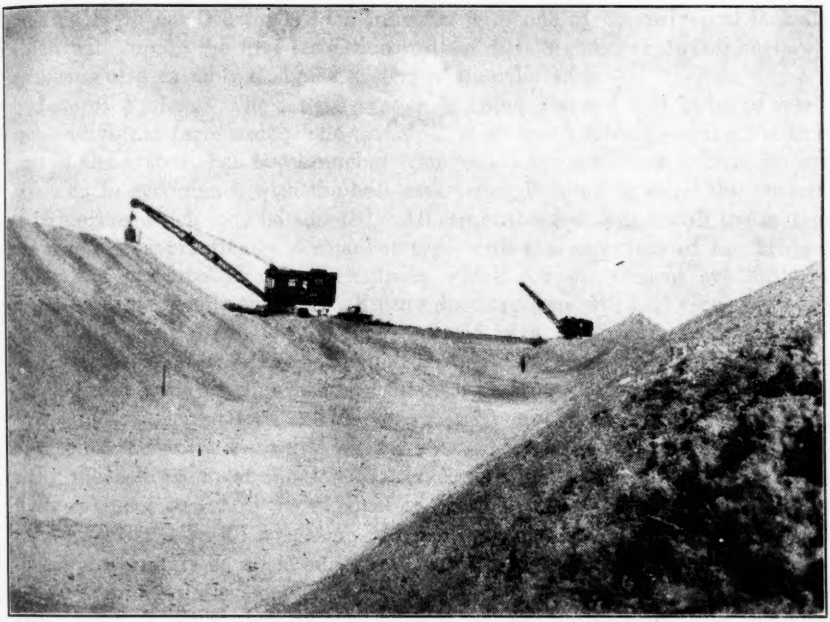

FIG. 8.—VIEW OF A SECTION OF THE FINISHED CANAL.

FIG. 9.—VIEW OF THE 5-METER CUT IN THE MAIN CANAL.

This main canal extends to a junction with one of the principal laterals which will supply the first irrigation unit of 40 000 acres, or 16 000 hectares, by means of a suitably designed system of distributaries.

Lateral System.—The lateral system is being planned and built to reach each individual farm unit. The principal (Camaron) lateral serving the first unit of the system, has two branches from which the necessary tributaries are extended in accordance with the best experience, keeping in mind the systems of irrigation which may be adopted. All structures connected with the system are made of a practically permanent type with the exception of the bridges across the principal drainage channels, which for the present are built of timber resting on pile supports. Future developments will best serve to determine which may ultimately best be replaced with more permanent material.

Drainage System.—The principal arteries of the drainage system are planned and built in connection with the irrigation systems. They are constructed of sufficient size and depth so as to provide not only for the safe disposal of the storm and waste waters, but also for the carriage, and more especially the collection, of the soil drainage. As these soils are generally of a fairly compact structure the removal of surplus waters is believed to be essential. Experience on all irrigation projects thus far has demonstrated that this collection can only be made effective by means of relatively deep ditches (7 to 10 ft.) whether the system is of the open or the closed type. In the project area considered, only open drains are built, for the reason that these will have to take care of quantitiies of storm waters, the volumes of which are known to be large but indeterminate. On account of the uncertainties as to the floods and the extent of future erosion, bridges across drains are being planned of wood with pile supports.

Construction of Works

Early in 1926 the Mexican Government entered into a contract with the J. G. White Engineering Corporation, S. en C., providing for the investigation and construction of a number of irrigation projects in different parts of the Republic. One of these projects, the Rio Mante, near Xicotencatl in the State of Tamaulipas, has been completed and is now in successful operation; another, the Calles Project in the State of Aguascalientes, on the Santiago River, is nearing completion; and the principal irrigation features of the Don Martin Project, including the distributing works for the first irrigation unit of 16 000 hectares (40 000 acres), will be available for the beginning of settlement and irrigation near the end of 1930. The Conchos Project in the State of Chihuahua, which has been given a most exhaustive line of investigation and study, is now under construction; it covers from 45 000 to 55 000 hectares (112 000 to 137 000 acres) of irrigable area, according to the scheme of development that may be adopted. Other projects in the course of construction and investigation are the Tula, in the State of Hidalgo, and the San Carlos, in the State of Coahuila. These two are old projects which require partial rehabilitation and the extension of the existing works constructed at an earlier date. The irrigation and settlement of the Don Martin Project require the construction and development of the following principal features now (1930) in course

of construction or well advanced toward completion in so far as they relate to the first irrigation unit: Rio Salado Dam; Rio Salado Auxiliary Dike; Canal System; Lateral System; Drainage System; and Roads.

Construction Operations

Río Salado Dam.—The length of haul of the material used in the earth and gravel section of the dam was about 2 km. (1¼ miles), and this haulage was effected by a railway system with seven to eight trains, drawn by 20-ton oil-burning locomotives. Each train consisted of from twelve to fourteen 5-yd. dump cars. Cars were loaded by two 2½-yd. steam shovels. Following each successive unloading, 30-h. p. tractors shifted the track a sufficient distance to permit the spreading of the materials, using two 60-h. p. caterpillar tractors and graders. After suitable sprinkling the material was compacted by four 12-ton road rollers in 8-in. layers. It is of ideal composition for a dam embankment of this type, consisting of about 65% of gravel well graded to form a compact mass, mixed with about 35% of "fines". In its compacted state the material weighs approximately 135 lb. per cu. ft.

As is usual in construction of this character, the surface was first stripped of all vegetable matter. Where heavy brush and undergrowth prevailed, the stripping was extended to include the root zone of this undergrowth.

The aggregate for the main body of the concrete overflow section was limestone obtained from the right abutment. Here, an opportunity was afforded for lengthening the spillway and the installing of four additional radial gates of the same size as the twenty-two automatic gates, but operated by direct control. The materials, suitably selected, were passed through a No. 6 gyratory crusher driven by a 240-h.p. Diesel engine of the marine type, the engine also furnishing the motive power for the screens, sand mill, mixers, and auxiliaries. After passing the crusher, the material was elevated 70 ft. to a revolving screen from which it passed into the storage bins used both for supplying the mixer plant below and for loading trucks. A part of the crushed material was returned to a 36-in. cone crusher which furnished the sand for the concrete, no natural sand or gravels having been available. Immediately below the storage bins was the mixing plant, which consisted of two 1-cu. yd. mixers from which the concrete was conveyed to the forms in side dump-cars. (See Figs. 10 and 11.)

The concrete for the outlet works and retaining wall was mixed by a system of from three to four 14 S mixers, of ½-yd. capacity. Side dump-cars and chutes were used as for the overflow section.

Auxiliary Dike.—Starting in December, 1927, the construction of the dike progressed uninterruptedly. The major part of the work was performed with three standard 36-in. elevating graders and from eight to twelve 1½-yd. dump wagons to each machine, according to the haul. Because of the shallow soil mantle suitable for this embankment the haul ranged from 55 to 180 m. (180 to 600 ft.). In order to expedite progress and to employ available forces of drivers and light-weight teams, the smaller dike sections were built with slip scrapers and mules. As with the main embankment, the material was spread, leveled in 8-in. layers, wetted, and then rolled with 12-ton road rollers.

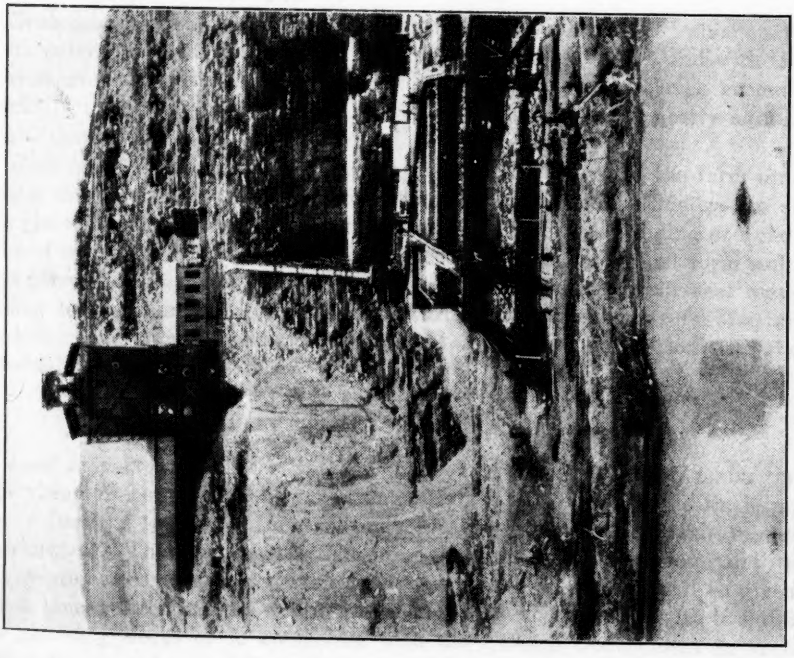

Fig. 10.—View of the Right Abutment of the Spillway Section.

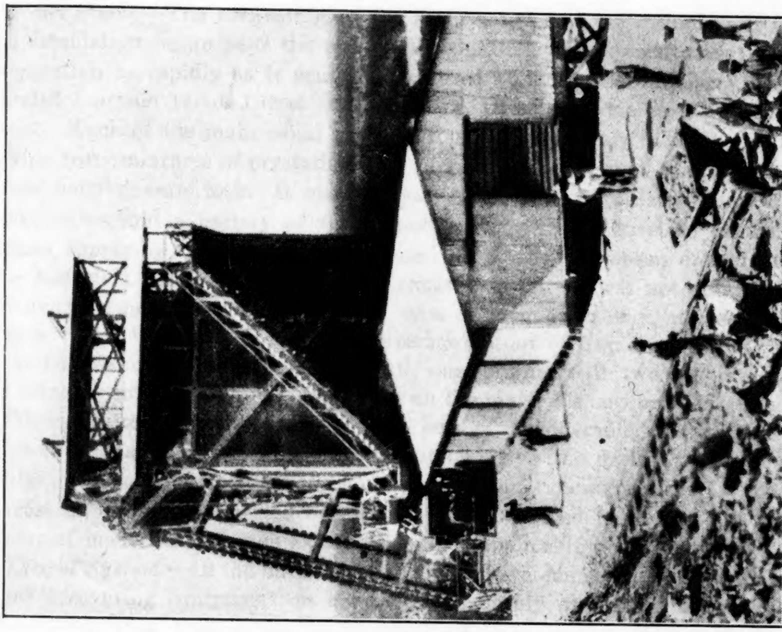

Fig. 11.—View of the Quarry at the Right Abutment.

Main Canal.—The excavation of the main canal and the stripping of the dam foundation began near the end of March, 1927. Equipment was placed in operation as rapidly as it could be procured from the manufacturers and included fourteen 1½-yd. Diesel and one 2½-yd. Diesel, electric drag-line excavators. Much of the main canal passes through uncommonly difficult ground ranging from mixtures of crystallized gypsum embedded in very compact clays to hard conglomerate rock. It was necessary, therefore, to use large quantities of explosives and a battery of from four to six portable gasoline air compressors supplying the necessary drills and auxiliaries to loosen and shatter these materials. As a result, the usual excavation output was not obtainable, although the operators were nearly all men of long experience and training in such work. Generally speaking, an average output of from 20 000 to 30 000 cu. m. (26 000 to 39 000 cu. yd.) per month was obtained with two 8-hour shifts and using 1½-cu. yd. machines, depending on the materials encountered. Much of the excavation had also to be made in cuts of considerable dimensions for which these machines were inadequate, but owing to the desire of the Commission of Irrigation to limit its equipment to such standard sizes as would be most useful in its expected operations, it was decided to choose only machinery of moderate size such as could be made applicable to future uses.

Lateral System.—In the building of the lateral system, advantage was taken of the excavating equipment on hand. The monthly excavation output was reduced 30% from that on the larger canals of 15 to 40-ft. bottom width and 4 to 8-ft. depth, in similar soils.

Drainage System.—The principal drains for disposing of soil drainage and storm waters are being built jointly with the distributing system and with the same excavating equipment, because of the relatively softer materials encountered. In these, the output and the unit costs have shown more nearly normal results than in the main canal system.

Road System.—Intimately connected with the laying out of the farm unit plan is the provision of an adequate system of public roads. This system as now planned and in process of construction provides for the building of a network of secondary or earth roads giving access to each individual farm unit. This network of secondary roads is united in a number of gravel surfaced roads leading to the principal highways and railroad stations, thereby facilitating marketing and shipments of produce as well as intercommunication with the principal population centers.

Settlement of and Re-Payment for Lands

Land Exchanges.—All the work on the Don Martin Project was under the supervision of the National Commission of Irrigation, created by the Enabling Act of January 4, 1926, to function under the Department of Agriculture and Development. This law provides that, upon the determination of the feasibility of any project composed of private lands, the owner or owners shall be given not to exceed three months within which they may voice their desire to build the necessary works or to co-operate with the Federal Government in such undertakings, under special conditions provided by this Act.

Omitting details, the law further provides that in case the Government builds the work covering private lands, it is to be reimbursed by a part of the owner's holdings such that its assessed unit value in the raw state multiplied by the area ceded to the Government shall be equal to the value of the area retained by the owner multiplied by the unit cost of reclamation or unit values arbitrarily applied. In making these adjustments, the Commission is authorized to add to the assessed value of the raw land in each project, a percentage so that the value of the irrigated land retained by the owner shall be equal to that of the property he possessed before the project was undertaken.

COLONIZATION

Mexican Citizens Favored.—The Commission desires above all things to make available the benefits of these irrigation developments to all Mexican citizens, many of whom live in adjoining States across the border where they have acquired considerable agricultural knowledge and experience. It is hoped that they may form the nucleus of a system of colonists, supplemented by others from within or without the confines of the Republic, the basic thought being to serve best a struggling population which is greatly in need of employment and home resources.

The National Commission of Irrigation has from the outset been conscious of the inherent difficulties attending the settlement of irrigation projects. For this reason every care has been taken to minimize the number of failures. To this end, the preliminary soil surveys upon which the outline of the project was based, were supplemented by a system of detailed soil surveys by competent experts. The precautions thus taken eliminate all doubt as to the adequacy of the soils or the system which is to furnish the water supply.

Following exhaustive studies of the problems of reclamation President Pascual Ortiz Rubio, under date of May 22, 1930, issued in effect the following resolution: That the first unit of the Rio Salado System shall be settled by colonists possessing the following primary requisites: They must be of Mexican nationality, have agricultural experience, be physically and morally capable, and have sufficient capital and resources to establish and support themselves on the land during the first year.

Further, that the Commission will select settlers according to the following preferences: (1) Original land owners within the limits to be colonized; (2) settlers or renters within the region; (3) Mexican farmers, including those outside of Mexican borders desiring to return; (4) Mexicans residing in cities who have the necessary primary qualifications; (5) alumni from the agricultural colleges who have completed their courses; and (6) persons not included within these items, but who have the requisite primary qualifications.

The prices to be paid for the irrigable lands cleared and ready for irrigation are 300 pesos per hectare ($54.50 per acre) for first class; 285 pesos per hectare ($52.00 per acre) for second class; and 270 pesos per hectare ($49.00 per acre) for lands of the third class.* To this may be added dry pasture land at the rates of 40, 15, and 7.50 pesos per hectare ($7.30, $2.74, and $1.37

* The values of 300, 285 and 270 pesos per hectare of irrigable land are based upon going values of similar lands within the Republic, subject to similar soil, climatic, and economic conditions.

per acre), according to classification fixed by the Commission. The colonist is to pay 5% of the cost price on signing the contract, the remainder in not to exceed twenty-five years; and, in addition, the annual operation and maintenance charges necessary for the upkeep of the system and 4% interest on the unpaid instalments.

With the listing of the applications by settlers it became increasingly apparent that a large number of otherwise desirable settlers found it difficult to make the initial payment of 5% of the construction payment, and also to meet the numerous additional items of expenses incident to the first season's operations. To meet this condition the President approved a resolution admitting colonists on a rental basis on a sale contract for a term of three years. The contract provides that the colonist pay the Government an annual rental of 20% of the gross crop returns for the use of the land and the irrigation system; 25% of the gross crop returns for the use of the irrigation system, including the furnishing of the necessary seeds; and 30% of the gross crop returns if the Commission also furnishes the necessary farm equipment, in addition to the seeds and the land as indicated.

It is to be understood that the value of these crops is to be credited to the colonist according to the market prices prevailing (the particular time is probably subject to future regulations by the Commission). Thus, if the value of the colonist's products should amount to 15 pesos per hectare ($2.74 per acre), and the cost of the water charges, including the rental, to 10 pesos per hectare ($1.83 per acre), the colonist is credited with the surplus 5 pesos ($0.91) on the construction charges. In other words, if the renter-colonist who furnishes his own seeds and implements gives his 20% of the gross produce, he is credited with the prevailing market price therefor from which is deducted the fixed charge of 5 pesos per hectare ($0.91 per acre) plus a charge of 50 centavos per 1 000 cu. m. ($0.64 per 100 000 cu. ft.) of water furnished to and measured at the land. Thus, an application of 60 cm. of water (2 ft. in depth) will cause a charge of 5 plus 3 pesos per hectare, making a total of 8 pesos per hectare ($1.46 per acre) to be deducted from his gross returns, the remainder to be credited to the construction charges applying to his lot.

The renter-colonist further obligates himself to cultivate not less than 50% of the irrigable land contained within his lot in a manner subject to the approval of and in accordance with the technical directions given him by the administrators of the local experiment farm. If he fails to accumulate a total of 10% of the value of the construction cost assessed against his land within the term of three years, his contract is then to be cancelled, and such payment benefits as may have accrued will be given to the succeeding settler.

Space does not permit of discussing the details many of which will have to be developed in the future as these and newer problems arise. The general statement gives one an idea of the extent the Mexican Federal Government is disposed to aid reclamation, which attitude appears to be generously supported by the Federal Congress.

The policies thus far pursued by the Presidents of Mexico since and including President Calles and so ably supported by the successive Commissions of Irrigation are deserving of the highest praise. Without a doubt they will

result in a material betterment of internal conditions. This should go far to halt emigration and the consequent neglect of the resources remaining undeveloped within the Republic, especially those relating to agriculture and the supplying of foodstuffs needed by its own people.

Authority and Direction of the Works

All the engineering and construction work described has been performed by the J. G. White Engineering Corporation, S. en C., under contract with the Government of Mexico and in conformity with the policies and general direction of the National Commission of Irrigation appointed by the President.

The original Commission appointed by President Calles was composed of Señores Francisco A. Salido, Director, and Javier Sanchez Mejorada and José Mares, Members. During the administration of President Portes Gil, the Commission was composed of Roberto Gayol, M. Am. Soc. C. E., and Señores Alfonso González Gallardo and González Robles, with Sr. Ignacio López Bancalari as Director General of the Commission.

The present Commission is composed of Gen. Manuel Perez Treviño, President (who is also Secretary of Agriculture and Development), Sr. Bancalari, Executive Director, and Sr. Leopoldo Vasquez, Secretary-Member of the Commission.

Mr. G. W. Caldwell is General Manager for the J. G. White Engineering Corporation, S. en C., and H. V. R. Thorne, Construction Supervisor, F. E. Weymouth, M. Am. Soc. C. E., was Chief Engineer until March 1, 1929, when he was succeeded by C. H. Howell, M. Am. Soc. C. E., all with offices in the City of Mexico.

The Commission's representatives on the project were Sr. Jesús Oropesa, Technical Supervisor, who was succeeded in April, 1929, by Sr. Manuel Bancalari. R. M. Conner, M. Am. Soc. C. E., is Superintendent of Construction; Sr. A. Becerril Colin was Assistant Superintendent until March, 1930, and the writer, Resident Engineer.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

GENERAL SPECIFICATIONS FOR STEEL RAILWAY BRIDGES

PREPARED BY CONFERENCE COMMITTEES FROM THE AMERICAN SOCIETY OF CIVIL ENGINEERS AND THE AMERICAN RAILWAY ENGINEERING ASSOCIATION

Discussion*

By Messrs. Otis Ellis Hovey, and Henry B. Seaman.

Otis Ellis Hovey,† M. Am. Soc. C. E. (by letter).‡—The first sentence of Article 300,§ reads, "the dead load shall be proportioned for unit stresses 50% greater than those allowed in Article 301." This loosely drawn sentence evidently means that the basic tensional unit stress for the dead load stresses shall be 16 000 lb. per sq. in., increased by 50%, or to 24 000 lb. per sq. in. The writer believes that this basic tensional unit stress is too great for use in designing new bridges to be built of ordinary structural carbon steel. In special cases in which the dead load is unusually great, the unit stress due to combined dead and live load with impact would approach 24 000 lb. per sq. in.

In its Final Report‖ the Special Committee on Stresses in Structural Steel recommended the use of a basic tensional unit stress of 20 000 lb. per sq. in. for buildings.

The Standard Specifications for Structural Steel for Bridges, Serial Designation: A7-29, of the American Society for Testing Materials, states that the tensile strength per square inch may vary from 55 000 to 65 000 lb. The yield point is specified to be not less than 30 000 lb. per sq. in. If this

* Discussion of the General Specifications for Steel Railway Bridges, continued from October, 1930, *Proceedings*.

† Asst. Chf. Engr., Am. Bridge Co., New York, N. Y.

‡ Received by the Secretary, October 11, 1930.

§ *Proceedings*, Am. Soc. C. E., December, 1929, Papers and Discussions, p. 2653.

‖ *Loc. cit.*, March, 1925, Papers and Discussions, p. 401.

steel is used, the writer prefers to limit the basic unit tensional stress to 18 000 lb. per sq. in. for bridges.

If and when the specifications for structural steel for bridges is changed to require an ultimate tensile strength of from 60 000 to 72 000, with a minimum yield point of about 34 000 lb. per sq. in., it would then become proper to specify a basic tensional unit stress of 20 000 lb. per sq. in. for such structures.

It is the writer's opinion that the basic unit stress should be a function of the physical qualities of the material which is used in a structure, and not of the kind of load which causes the stresses. In a railway bridge—if the probable future live load is used when calculating the stresses, and the dynamic increment, called impact, is added, together with the dead load stresses—the unit stress to be applied to the combined stress should be that which is most appropriate for the steel used in the structure.

HENRY B. SEAMAN,* M. AM. SOC. C. E. (by letter).†—The clause which permits the use of a special stress for dead load is the connecting link between short-span and long-span bridges. For short spans it is unimportant, but for long spans it is indispensable. The dead load is definite in amount and effect, while the live load is variable and uncertain.

About twenty-five years ago, when the use of steel was in its infancy and comparatively untried, 20.0 kips per sq. in. was considered the highest stress to which a bridge should be maintained, and the permissible increase would have been 25%; that is, from 16.0 kips to 20.0 kips; but the more recent practice, justified by an experience of years of maintenance, has placed 24.0 kips well within safe limits.

It should be borne in mind that the extreme stress is only attained by a full overload, 50% greater than that for which the bridge is designed; and that for spans in which the dead and live loads are equal, say, 300 ft., or more, in length, the greatest stress under the specified loading would not exceed 20.0 kips per sq. in., and for the unloaded bridge the stress would not exceed 12.0 kips per sq. in. For shorter spans, the dead load stresses would be even less. Engineers cannot now return to the days when only 10.0 kips per sq. in. was allowed because of inexperience, but must move forward according to the light which that experience affords.

If 20.0 kips, instead of 24.0 kips per sq. in., were now adopted as the limit of allowable stress, it might mean the financial embarrassment of many railroads which have long maintained their bridges at 26.0 kips per sq. in., and would be considered culpably negligent if they were to continue to do so after 20.0 kips per sq. in. were adopted as the safe limit for dead load stress.

The ultimate strength of the material will average more than 60.0 kips per sq. in., and the yield point more than 35.0 kips, although specified to be not less than 30.0 kips per sq. in.

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* Cons. Engr., Brooklyn, N. Y.

† Received by the Secretary, December 1, 1930.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

PLASTIC FLOW IN CONCRETE ARCHES

Discussion*

By Messrs. G. C. Staehle, and A. Floris

G. C. Staehle,† Assoc. M. Am. Soc. C. E. (by letter).‡—In a study of the effects of time yield or plastic flow upon concrete arches the following items are of outstanding importance:

(*a*) The modulus of elasticity of concrete, as determined by the ordinary quick loading of laboratory practice, increases progressively with age over a period of several months or more.

(*b*) The modulus decreases with age under sustained loading,§ the amount and rate of decrease being functions of the ratio of applied unit stress to ultimate unit strength.

(*c*) The rate of advancing deformation with time for concrete under sustained loading is also a function of the ratio of applied unit stress to ultimate unit strength. (Experiments‖ by G. A. Maney, M. Am. Soc. C. E., indicate that, within the limits of working stress, the increase of deformation with time, commonly attributed to sustained load, is, in reality, due principally to shrinkage.)

(*d*) Repeated applications of stress, within the ultimate strength, each unloading being followed by a period of rest—even for a very short period—raise the proportional limit of concrete in much the same manner as that of steel and other metals. A plausible and very interesting explanation of this phenomenon has been previously given.¶ The stress-strain relation thus becomes a linear one up to a relatively high percentage of the ultimate strength for this condition.

Bearing these four major points in mind, the ratios of dead load to total load, and of dead load and total load to ultimate strength, have a definite bearing on the amount of plastic flow which may be expected to occur in any particular structure.

* Discussion of the paper by Lorenz G. Straub, Jun. Am. Soc. C. E., continued from September, 1930, *Proceedings*.

† With Portland Cement Assoc., Chicago, Ill.

‡ Received by the Secretary, October 11, 1930.

§ *Proceedings*, Am. Soc. for Testing Materials (1929), p. 695.

‖ *Engineering and Contracting*, June, 1930, p. 234.

¶ *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 890.

It should also be borne in mind that plastic flow is accentuated by rapid drying and retarded by moist curing. This effect is influenced, in turn, by the initial wetness of the concrete mixture. Excessively wet mixes are, indubitably, responsible for the majority of extreme cases of plastic flow. Delayed hardening, due to freezing or to low temperatures, and poor aggregates are responsible for most other cases.

The writer's attention was first called to the noticeable effects of plastic flow about seventeen years ago. Since then he has encountered several other extreme cases, but in practically every instance has been able to trace the condition back to one of the three causes mentioned.

Concerning the many statements found in engineering literature as to the inelastic properties of concrete, the writer, in his own experiments, has found that, generally speaking, for initial and repeated applications of stress, perfect elasticity is obtained in ordinarily good concrete up to 40% of the ultimate strength at the age tested. This 40% is a minimum value based upon concretes having a nominal 28-day strength of 2 000 lb. per sq. in., and more, and applies from ages as early as 7 days for concrete having a nominal 28-day strength of 3 000 lb. per sq. in., and more. For 2 000-lb. concrete, it has been found to apply at an age as early as 14 days.

He has also found that linear relation of stress to strain up to a very high percentage of the ultimate strength may be obtained with the higher strength concretes on initial as well as on repeated applications of stress, the percentage decreasing with the strength of the concrete, but remaining well outside the range of the working stress for concretes having a nominal 28-day strength of 2 000 lb. per sq. in., and more.

Although 28 days is commonly accepted as a criterion in the adoption of working stress and elastic properties for design purposes, consideration should also be given to the increase of modulus with age and the coincident raising of the proportional limit during the process of hydration.

Arguing from these premises, it becomes questionable as to whether or not the author's attempt to provide for a variable stress-strain relationship in the design of concrete arches is of practical value.

A. Floris,* Esq. (by letter).†—Although the use of masonry arches in structures is very old, the correct interpretation of their statical behavior is of a comparatively recent date. In the development of their theory the French and German engineers took a leading part.

The introduction of the elastic theory in the analysis of masonry arches, based upon Hooke's law as an approximation, was a marked advance in the design of arches. This theory satisfied the requirements of practice, as long as the application of concrete arches to long-span bridges was somewhat restricted. However, the use of arches with great spans is increasing steadily and a more elaborate analysis of their stress condition is highly desirable.

It is very gratifying, therefore, that the author, in his pre-eminently able analysis indicates further improvements in the design of concrete arches,

* Civ. Engr., Los Angeles, Calif.

† Received by the Secretary, October 17, 1930.

which are of a distinct and practical value. Basing his thesis on the Bach-Schüle exponential law of elastic deformation and deriving from available experimental data an exponential law for the plastic flow of the concrete under sustained stress, Mr. Straub builds a new theory of arches which is a decidedly better approximation to the actual stress condition than the method founded on the linear distribution of stress, which disregards the time element.

It cannot be expected that empirical formulas such as those used in the paper will be perfect. They can be improved, however, as more information on the behavior of the concrete under stress accumulates. The remaining theoretical part of the author's analysis, being based upon the logical application of the laws of mechanics, is accurate and sound.

In the near future, perhaps, further progress in the stress analysis of concrete arches will be made, but the credit of first developing a more exact theory of concrete arches belongs rightly to the author. The paper deserves the closest attention of the students of concrete arches.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

LAMINATED ARCH DAMS WITH FORKED ABUTMENTS

Discussion*

By Fred A. Noetzli,† M. Am. Soc. C. E.

Fred A. Noetzli,‡ M. Am. Soc. C. E. (by letter).§—There are several more or less independent features of arch dam form and layout described in this paper, namely, the so-called "forked abutments", a pronounced sloping of the crown of the arch, and the subdivision of the arch, where indicated, into two or more arch laminæ. Furthermore, a simple and convenient method is given whereby the arch stresses at the crown and the abutments, due to uniform water pressure or change of temperature, can be computed from the axial stress (cylinder formula) by mere multiplication with a coefficient to be taken from diagrams given in the paper. Similar diagrams are given for coefficients computed by the theory of "thick" arches.

The three features described in the paper may be used either singly, entirely, or in combinations of two. An arch slightly overhanging in its upper central portion will be found more economical in most cases and will have a better distribution of stresses than an arch dam of the ordinary type.

Forked abutments on one or both sides of the arch will be advantageous at most dam sites. As mentioned by Mr. Hanna,‖ they have been used with success in the Santeetlah and Calderwood (arch) Dams; and the Stewart Mountain (arch) Dam on the Salt River, in Arizona, has forked abutments.

The writer agrees with Mr. Jorgensen¶ that in fairly thin dams the arch should not be laminated. The greatest benefit from laminating is obtained in a relatively thick structure. To the extent that they are effective, the laminations tend to give a better stress distribution in the vertical cantilever elements as well as in the horizontal arches. They also improve the distribution of the

* Discussion of the paper by Fred A. Noetzli, M. Am. Soc. C. E., continued from October, 1930, *Proceedings*.

† Author's closure.

‡ Cons. Hydr. Engr., Los Angeles, Calif.

§ Received by the Secretary, October 28, 1930.

‖ *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1724.

¶ *Loc. cit.*, May, 1930, Papers and Discussions, p. 1107.

foundation pressures both under the vertical cantilevers and, laterally, at the arch abutments.

Mr. Jorgensen and Mr. Hanna* appear to place considerable reliance on shear resistance for carrying the load in the lower portions of an arch dam. Consideration should be given in this connection to the fact that, at the support of an ordinary cantilever or beam of plain concrete, the shear stresses may be relatively small while the compression or tension stresses from bending may be large enough to produce failure. Small shear values alone, regardless of their magnitude, can not be relied on to insure the stability of a structure, such as a beam, a cantilever, or an arch, unless the tension and compression stresses are also within proper limits.

It is gratifying that, by a different process from the one used by the writer, Professor Cain† arrived at the same equation from which the coefficients of Figs. 1‡ and 2§ were computed. A further check is furnished by comparison with Figs. 3‖ and 4¶ for "thick" arches. The writer is in full accord with Professor Cain regarding the desirability of vertical radial contraction joints in arch dams. Such vertical joints were purposely omitted in the Stevenson Creek Experimental Arch Dam for the main reason that it was desired to test horizontal arch elements which would be continuous from abutment to abutment so as to permit the measurement of continuous strain curves between abutments without the presence of disturbing irregularities at such joints. Although in the course of the experiment several cracks developed in the dam—particularly a vertical crack in the center extending about 10 ft. from the crest downward and another crack in the same vertical plane extending from the base upward for a distance af about 13 ft.—fortunately none of these crossed the horizontal strain-gauge line at Elevation 30. The continuity of the strain-gauge line at this elevation, therefore, remained unimpaired. The measurement of strains along this line furnished some of the most interesting results of the Test Dam investigation.**

There is apparently a misunderstanding regarding the meaning of the term, "rib-shortening" stresses, as used in the derivation of Equations (1) to (5)†† for the total stresses resulting from the shortening of the arch rib, that is, for direct and bending stresses combined. Professor Cain's Equation (12)‡‡ gives the direct stresses only.

Mr. Jakobsen‡‡ and Dr. Vogt§§ present interesting computations relative to the effect of friction in the joints between the arch laminæ. A certain amount of friction will be unavoidable. It is evident, however, that the worst that could happen is that the friction would prevent all movement. The structure

* *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1722.

† *Loc. cit.*, May, 1930, Papers and Discussions, p. 1108.

‡ *Loc. cit.*, February, 1930, Papers and Discussions, p. 267.

§ *Loc. cit.*, p. 268.

‖ *Loc. cit.*, p. 270.

¶ *Loc. cit.*, p. 271.

** Report on Arch Dam Investigation, *Proceedings*, Am. Soc. C. E., May, 1928, Part 3, p. 164.

†† *Proceedings*, Am. Soc. C. E., February, 1930, Papers and Discussions, p. 265.

‡‡ *Loc. cit.*, May, 1930, Papers and Discussions, p. 1111.

§§ *Loc. cit.*, August, 1930, Papers and Discussions, p. 1450.

then would act simply as a solid arch. However, any movement in a joint would tend to diminish the arch stresses. The extent of this re-adjustment of stresses is in proportion to the tangential movement in the joints. In this connection it is of importance to note that deformations of a laminated arch due to shrinkage and changes of temperature involve very small radial stresses and, consequently, develop very little, if any, friction in the joints. For such deformations, therefore, an adjustment of the stresses could take place approximately as indicated by theory. If water pressure, shrinkage of concrete, and change of temperature of the arch occur in combination with each other, as must usually be assumed for the critical stress conditions, the radial pressure upon the joints remains practically the same as for water pressure alone, while the shear, under certain conditions, may be doubled or tripled, and, accordingly, also the tendency of sliding in the joint.

Mr. Jakobsen recommends* the grouting of contraction joints to counteract the shrinkage of the concrete and to improve the stress conditions in arch dams. He states that "this method has been used in connection with a number of dams and apparently with considerable success." The writer has had occasion to get first-hand information on, and, in a few cases, to be directly connected with, such grouting operations on several dams. This experience leads him to believe that the result of grouting of contraction joints in arch dams is very uncertain, to say the least. There is the danger of some portions of the area of a joint being grouted and others not, which results in concentrations of pressures over the grouted areas when the dam is loaded. In the Pacoima (arch) Dam the quantity of cement grout forced through the grout pipes into the contraction joints under a pressure of 100 lb. per sq. in. was less than what would have been required for filling the pipes. This indicates that some of the pipes were at least partly clogged by "fines" or other materials during the construction of the dam. Under such conditions it is very doubtful just how much grout was actually forced into the joints themselves and how it was distributed over the section.

The writer agrees with Mr. Werner* that it is important to secure monolithic action in the forked abutments. Steel reinforcement placed cross-wise between the buttress and the gravity wing, as indicated in Fig. 7,† will promote such monolithic action. Some extra concrete placed in the trough between the buttress and the gravity wing will further improve it. In order to avoid the disadvantages due to shrinkage of the concrete, as pointed out by Mr. Werner, it will be advisable to keep the buttresses as low and as short as possible. As far as the writer knows no difficulty has ever been experienced in arch dams abutting against gravity tangents.

Dr. Vogt refers‡ to theoretical and experimental deductions which indicate that the thrust of the uppermost arches is inclined at the abutments.

Since the crown of the dams shown in the paper is sloping, the actual load-carrying arch elements will be inclined downwardly at the abutments consid-

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 1115.

† *Loc. cit.*, February, 1930, Papers and Discussions, p. 281.

‡ *Loc. cit.*, August, 1930, Papers and Discussions, p. 1447.

erably more than in the dams investigated by Dr. Vogt. This is one of the important features of the design as it permits with safety the use of higher forked abutments and consequent shorter radii and spans in the central arch. The most economical layout is best determined by trial. For the uppermost arches it is of advantage to use a relatively long radius and a relatively small central angle so as to direct the arch thrust "into" the hillside rather than parallel to it, with consequential beneficial results as regards the stability of the forked abutments. For practical reasons these arches are thicker than would be theoretically necessary, and, therefore, the central angles need not be very large to give stresses within permissible limits.

Regarding materials for decreasing friction in the joints between the arch laminæ the writer is emphatically against the use of any substance which—either because of the composition or because of the thickness of the layer—might flow to a considerable extent. He believes that a thin coat of asphaltic paint or some of the other materials mentioned in the paper will be sufficient to enable movement in the joint leading to at least a partial re-adjustment of the arch stresses with consequent beneficial results. The writer is grateful to Dr. Vogt* for having emphasized the probability of the up-stream laminæ being stressed relatively more and taking a larger portion of the load than the down-stream laminæ. Dr. Vogt gives as reasons the expansion of the up-stream laminæ due to water-soaking, and a smaller value of a "sustained modulus of elasticity" for continuous load on the dry lower laminæ. In a "thick" arch of a dam the highest stresses occur at the abutment intrados. Along the entire extrados the stresses are relatively small. As an example, consider the arch assumed by Mr. Jakobsen† for which $\frac{t}{r} = 0.50$ and $2\phi = 120$ degrees. Assume this arch to be stressed by an axial compression of 250 lb. per sq. in. (cylinder formula). From Figs. 3 and 4, the coefficient, K, for the given values of $\frac{t}{r}$ and 2ϕ, was obtained for use in computing the arch stresses. The cylinder stress, multiplied with the values, K, gives the stresses in the arch at the crown and abutment directly, as shown in Table 3.

TABLE 3.—COMPUTATIONS FOR DETERMINING ARCH STRESS.

Point of arch.	Axial stress, in pounds per square inch.	K.	Arch stress, in pounds per square inch.
Extrados: Crown..................	250	1.08	270 (compression)
Extrados: Abutment..............	250	−0.38	−95 (tension)
Intrados: Crown..................	250	0.12	30 (compression)
Intrados: Abutment..............	250	2.52	630 (compression)

Evidently, in the solid arch the concrete near the extrados is used very inefficiently, the stress being 270 lb. per sq. in. compression at the crown and

* *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1450.

† *Loc. cit.*, May, 1930, Papers and Discussions, p. 1112.

95 lb. per sq. in. tension at the abutment, while the greatest stress in the arch is 630 lb. per sq. in. compression at the abutment intrados. Consequently, if subdividing such an arch into two or three laminæ tends to put more load and a correspondingly greater stress on the up-stream arch for the reasons given by Dr. Vogt, the results can be only beneficial. If, in this example, a drop of temperature and shrinkage of concrete were considered, the comparison would be still more strikingly in favor of the laminated type.

If a reliable method of grouting of contraction joints of arch dams were available the writer would recommend that the joints in the up-stream arch be grouted under a higher pressure than those of the down-stream arches. As a result, the up-stream arch would take more stress and, therefore, carry more load than the corresponding parts of the solid arch. Naturally, certain other parts of the arch would be relieved by the same amount of load. Some over-stressing of the up-stream arch within reason would be beneficial rather than detrimental to the safety of the structure as a whole, inasmuch as it would lead to a better distribution of the foundation pressures because the thrust of the up-stream arch would engage the up-stream portions of the side abutments which in the solid arch are stressed very little and may even be theoretically under tension. It also should be borne in mind that there is not the remotest possibility of independent failure of the up-stream arch because it could not break through without deflecting and thereby engaging the down-stream arches. This, of course, would relieve the up-stream arch immediately of the assumed excessive load.

Mr. Bauman* shows by the ellipse of elasticity the greater degree of flexibility of a laminated arch as compared to that of a solid arch, even if the joints should be only 50% effective. Greater flexibility results in a decrease of the bending stresses due to rib-shortening, temperature drop, and shrinkage, and a corresponding greater efficiency of the arch. Mr. Bauman suggests the possibility of slips between the laminæ. For the arch shown in Fig. 8† a maximum movement of 0.06 in. was computed for a change from no load to full load. It needs a rather vivid imagination to speak of "dam quakes" for such small movements.

Mr. Hanna‡ gives an interesting contribution to the discussion in the description of several arch dams in which some of the characteristics of the type of dam described by the writer have been used to advantage. He also calls attention to the feasibility of an overhang of the arch in an up-stream direction adjacent to the abutments. Mr. Hanna expresses doubt regarding the proper transmission of loads across the lamination joints. In this connection, it is of importance to consider that the second lamina will be cast tightly against the first which previously may have been built one or two lifts higher. Before pouring the second lamina the adjoining face of the first will be painted with asphalt so as to prevent bonding and decrease friction. Nevertheless, the two laminæ will be in direct contact with each other at all points. The effect of shrinkage of the concrete in a transverse direction may tend slightly to

* *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1721.
† *Loc. cit.*, February, 1930, Papers and Discussions, p. 282.
‡ *Loc. cit.*, September, 1930, Papers and Discussions, p. 1724.

open the joint, while on the other hand the overhang of the dam will have the tendency to keep it closed.

Assume the water to rise slowly in the reservoir. It will exert a uniform pressure against the up-stream lamina. In the case of two laminæ in contact with each other both will deflect equally. If the deflection lines of two arches are of practically identical shape (as are the up-stream and the down-stream laminæ of an arch dam), the loads causing these identical deflections must also be practically identical. The water load on the arch, therefore, is divided equally between the two laminæ. If, as assumed by Mr. Hanna, the down-stream arch deflected over a certain distance more than the up-stream arch, the joint would open naturally. As no load can be transmitted across an open joint the up-stream arch would have to carry the total water load, while the down-stream arch would be without load over this distance. Two or three determinations of arch deflections under these assumptions will quickly convince even the most skeptical that there is no possibility of such conditions arising. Therefore, if the joint does not open, the deflections of the two arches —and also the loads and stresses—must be identical, at least within the limits of similarity between the up-stream and the down-stream arch. The writer, therefore, is unable to agree with Mr. Hanna* that "the loads on the secondary arches will have different distributions and the deflections of the secondary arches will not conform accurately to those of the primary arch and to each other."

Mr. Eremin† cites Professor Timoshenko's equation for the maximum pressure beyond which an arch might buckle. Thin dams in which $\frac{t}{r}$ is not more than, say, 0.15, require no laminating and in thick dams there will seldom be need of more than one or two joints, so that the question of buckling will scarcely ever arise in this type of structure.

* *Proceedings,* Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1725.
† *Loc. cit.*, October, 1930, Papers and Discussions, p. 1897.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS
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RELATION BETWEEN RAIL AND WATERWAY TRANSPORTATION: A SYMPOSIUM

Discussion*

By Henry W. Hobbs, M. Am. Soc. C. E.

Henry W. Hobbs,† M. Am. Soc. C. E. (by letter).‡—The discussion of this subject has raised the question of the economic justification of inland waterway improvement at Federal expense. Before the construction of railways, rivers and canals provided the best, and in many cases the only, means of transportation. They continued to be the primary means of transportation up to about 1860, the early railroads acting mainly as feeders to the waterways. With the extension of the railroads severe competition developed between them and the waterways, as well as among the different rail lines. The effect of this competition is still seen in the rail rate structure of the country, with its marked discrimination in favor of points with water competition. The improvement of these earlier waterways was fully justified, as they formed the primary means of transportation and were important factors in the development of the United States.

In the latter half of the Nineteenth Century there was a period when many waterway improvements were authorized, some of which were of questionable value. To-day, waterway legislation is probably subjected to more careful study than any other question coming before Congress. In 1902, Congress created the Board of Engineers for Rivers and Harbors, to which all projected waterway improvements are referred for study and recommendation. Each project is now submitted for study to an officer of the Corps of Engineers who is stationed in the general locality, where a district office is maintained. He first submits a preliminary examination report, which contains such economic data as he is able to collect, with a general description of the locality and the

* Discussion on the Symposium on Relation Between Rail and Waterway Transportation, continued from September, 1930, *Proceedings*.

† Executive Secy., Board of Engrs. for Rivers and Harbors, Washington, D. C.

‡ Received by the Secretary, November 1, 1930.

details of the improvement desired. This report is then acted upon by another officer of the Corps of Engineers, who, as Division Engineer, has charge of a number of districts. The District Engineer's report, with the comments of the Division Engineer, is next studied by the Board of Engineers for Rivers and Harbors. If it finds that the proposed improvement appears economically justified, a field survey is made to determine the cost of the work and such other features as appear necessary in reaching a final conclusion. The survey report is made by the District Engineer, receives the comment of the Division Engineer, and the case is again considered by the Board of Engineers for Rivers and Harbors.

In the office of the Board, engineers, statisticians, economists, and rate clerks assist in working up the information received from the District Engineer and from other sources. This includes data concerning the potential commerce and the points of origin and destination of the several commodities. Much information of this character is obtained from the Department of Commerce. The practicability of the expected movements is studied, together with the available means of shipment and the transportation costs. The prospective savings in transportation costs are the differences between the existing rates on the same commodities, between the same or equivalent points, and the cost by an improved channel. The transportation costs by water include all terminal and transfer charges. An unfavorable report is indicated if these savings are not sufficient to show a considerable margin above the annual carrying charges on the proposed improvement. These charges include interest and amortization on the investment in channel improvement, and the maintenance cost, including the operating cost of locks and dams if such works form part of the project. No account is taken of savings to the public resulting from any possible rail-rate reduction to water competitive points. The principal traffic of the waterways is made up of bulk commodities, and little consideration is given in the Board's studies to the movement of high-grade general freight. About two-thirds of the recommendations made by the Board to Congress are unfavorable.

The claim that water transportation is more expensive than rail appears to be based on a comparison of water-haulage costs, including carrying charges on the improvement, with the average revenue per ton-mile of the railroads serving the territory. Taking into consideration the difference in the character of the shipments by water and by rail, and particularly taking account of the many peculiarities of the rail rate structure, it is apparent that such a comparison does not give a correct result. It appears that the only fair way to arrive at the saving in moving a particular commodity by water rather than by other means is by a direct comparison of the rates on that particular commodity between the same or equivalent points. To illustrate this, the average cost of moving coal on the Monongahela River is about 3 mills per ton-mile, while the rail rate on coal on parallel roads, for the same average haul, is 19 mills per ton-mile. This ton-mile cost on the Monongahela River cannot be used on another stream without making corrections for channel dimensions, current velocity, distances between the origin and destination of the commodity, size of cargoes, regularity of shipments, back haul, and other

conditions peculiar to the river under consideration. Wheat is moved by the Inland Waterways Corporation from St. Louis, Mo., to New Orleans, La., at a rate of 2 mills per ton-mile, or $1.30 per ton under the rail rate.

The cost to the people of the United States of making waterway transportation possible is covered by taxation. While the total taxes in the United States have increased greatly since 1923, Federal rates of taxation have been several times reduced during the same period. The returns which the people receive from this taxation for waterways come in the form of reduced commodity prices. The savings to industries, due to the lower cost of transporting raw materials by water, make possible the marketing at lower prices of the products of the industries affected. The iron and steel industry may be noted as one of those affected. These commodities enter into the construction of many things in general use, such as household utensils and equipment, thereby distributing the benefits to the entire population.

The cost of moving coal on the Monongahela River has been cited. The annual carrying charges on the improvement of the Monongahela River, based on the ton-mileage of 1929, are less than 1¼ mills per ton-mile. The carrying charges on the improvement of the Ohio River are greater than this at the present time, but will decrease as the traffic increases. Taking the Ohio River system as a whole, the total ton-mile movement in 1929 was 2 886 436 000. The savings on the Monongahela River alone are in excess of the annual charges for the entire Ohio River system. The Ohio River improvement, although only recently completed, already shows returns slightly in excess of the annual costs. The traffic on the system in 1929 totaled 59 537 000 tons, an increase of about 10 000 000 tons over that for 1925. The activities of large concerns, which have expended many millions of dollars in building towboats and barges, indicate that the movement of commerce on the Ohio River system will continue to increase. The total commerce on rivers and canals in 1929, exclusive of the Great Lakes and not including rafted timber, was 170 688 000 tons, an increase of about 35 000 000 tons over 1925. The ton-mileage was 9 446 712 000; in 1925, it was 8 417 052 000.

After a waterway has been completed and has been in operation a sufficient length of time to indicate its value as a part of the transportation system of the country, careful studies are made to determine the justification for the expenditures made by the Federal Government. In a few cases negative results have been shown. The Cape Fear River above Wilmington, N. C., has not shown a profit, as the improvement does not extend far enough up the river to give adequate depth at Fayetteville, the center of the principal tributary area. The St. Johns River, in Florida, between Jacksonville and Palatka, shows a small deficit, although the section of the same river between Palatka and Lake Harney shows a large profit. The only stream on which a considerable deficit has been shown is the Warrior River, in Alabama. Water transportation on this stream was handicapped for a number of years by an unfair division of rates between the rail and water carriers. Considered as a whole, the waterways of the country return each year, in reduced transportation costs, a sum much greater than the carrying charges on the improvements.

Dependability of service is an important matter. The length of the navigable season on the Great Lakes is about eight months. During the closed season it has not been found necessary to continue shipments of the principal commodities by rail. Large quantities of these bulk commodities are stored during the navigation season to preclude the necessity for such movements at higher cost during the closed season. The Ohio River is now open to navigation throughout the year, being subject in general to unimportant delays averaging not to exceed ten days annually. Such delays are not of serious importance on transportation lanes carrying principally bulk commodities, and instances can be cited where delivery is made more promptly by water than by rail.

Some classes of freight move most cheaply by water, others by rail, and others by motor truck. The greatest economy of transportation can be obtained by a proper co-ordination of these various means of transportation. It would be an economic loss to do away with the cheaper forms of transportation, whether it be by waterway, by railroad, or by truck, in favor of movement at higher cost over any other of these facilities. An ideal condition would provide for shipments, by the various routes, of the particular commodities best suited for transportation over each.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

FREQUENCY AND INTENSITY OF EXCESSIVE RAINFALLS AT BOSTON, MASSACHUSETTS

Discussion*

By Messrs. C. S. Jarvis, and Milton F. Wagnitz and Lewis C. Wilcoxen

C. S. Jarvis,† M. Am. Soc. C. E. (by letter).‡—The author has set forth in summarized form some of the outstanding meteorological data with which a designer of waterways should be concerned. Only after a thorough analysis and concentration, comparable with the well-known process developed by the mining industry, will the greatest values of such records be made available. While discoveries or observations locate and define the immense volumes in which values may be found, the processes of reduction and separation bring forth the precious elements from the inert mass.

Some of the quantities displayed in Fig. 1§ are re-arranged in Table 3.

TABLE 3.—Maximum Rainfall Intensities at Boston, Massachusetts.
(Expressed in inches per hour.)

Rainfall periods.	Frequencies, in Years.							
	1.	2.	5.	10.	16⅔.	25.	50.	100.
5 min.	2.8	3.3	4.3	5.3	5.8	6.7	8.1	10.0
15 min.	1.85	2.3	2.8	3.4	3.8	4.3	5.4	6.5
60 min.	0.83	1.0	1.3	1.6	1.75	2.0	2.4	2.9
3 hours	0.40	0.48	0.63	0.75	0.84	0.98	1.20	1.50
24 hours	0.10	0.13	0.16	0.18	0.20	0.24	0.27	0.34

This table shows that the maximum intensity to be expected during a century is about double that of a 10-year period, or four times that of yearly frequency. Conversely, a 10-year maximum intensity is to that of a 1-year fre-

* Discussion of the paper by Charles W. Sherman, M. Am. Soc. C. E., continued from September, 1930, *Proceedings*.

† Prin. Hydr. Engr., U. S. Engr. Office, War Dept., Washington, D. C.

‡ Received by the Secretary, August 7, 1930.

§ *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 722.

quency as a century maximum is to a 10-year maximum. Furthermore, a 50-year maximum is approximately double that of a 5-year maximum; and, similarly, for any other periods differing by a factor, 10. The logical assumption from the foregoing is that the 1 000-year maximum would be double the intensity listed in Table 3 recorded for a century, for each designated period, whether for minutes, hours, or for a day.

Examining the various columns of Table 2,* it is apparent that the summations of maximum rates for the ten recorded storms during 15, 30, 45, 60, 120, and 180-min. intervals, respectively, are nearly 5.5, 8, 9, 9.5, 10, and 6.5 times greater than for the equal periods immediately preceding the intense rainfall. This is quite in accordance with the usual rainfall habits in temperate zones. The first few drops serve as a warning even though the sky may be bright or only slightly cloudy overhead. As the storm cloud advances the curtain of rain often inclines at a considerable angle, or whips about similarly to window draperies. Later, the brief intermittent squalls are displaced either by a steady downpour, or by a period of calm succeeding the storm, depending on the location of the atmospheric disturbance. its duration, and the path described.

It is re-assuring to find some semblance of consistency even among habits proverbially regarded as fitful and uncertain, such as those displayed by rainfall. The author has performed a lasting service by his presentation of the subject.

Milton F. Wagnitz,† M. Am. Soc. C. E., and Lewis C. Wilcoxen,‡ Assoc. M. Am. Soc. C. E. (by letter).§—This paper is of particular importance. It establishes a basis by which the rainfall characteristics of different parts of the United States may be compared, and paves the way for determining the general laws of rainfall.

The writers' discussion is limited to the analysis of the rainfall data of Detroit, Mich. Before submitting the results for comparative purposes, an explanation of these data will be made. The statistical method used will also be outlined because it is believed that, for comparative purposes, the more exact results obtained are decidedly worth the labor. The rainfall data used are those of the U. S. Weather Bureau Station, situated on the Majestic Building, in Detroit. The records cover the 30-year period from 1896 to 1925, inclusive.

Records of excessive storms were tabulated by standard methods, that is, the period rates of each storm were tabulated, beginning with the period of maximum intensity. The periods taken were 5, 10, 15, 20, 30, 45, 60, 80, 100, 120, 150, 180, 210, 240, 270, 300, and 360 min. Intensities which were lower than those of succeeding periods were tabulated on the principle of extended duration, that is, the period of low intensity was credited with the rate of the higher succeeding period. The 1-year storm was taken as that hypothetical storm, the period intensities of which were equalled or exceeded thirty times. The 2-year storm is the one the period intensities of which were equalled or

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 725.

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‡ Asst. Civ. Engr., City Engr.'s Office, Detroit, Mich.

§ Received by the Secretary, October 31, 1930.

exceeded fifteen times, etc. The hypothetical frequency storms determined on this basis are shown on Fig. 3. It will be seen that the higher frequencies become progressively irregular. This, of course, should be expected since data by which they are determined, decrease in proportion to the frequency.

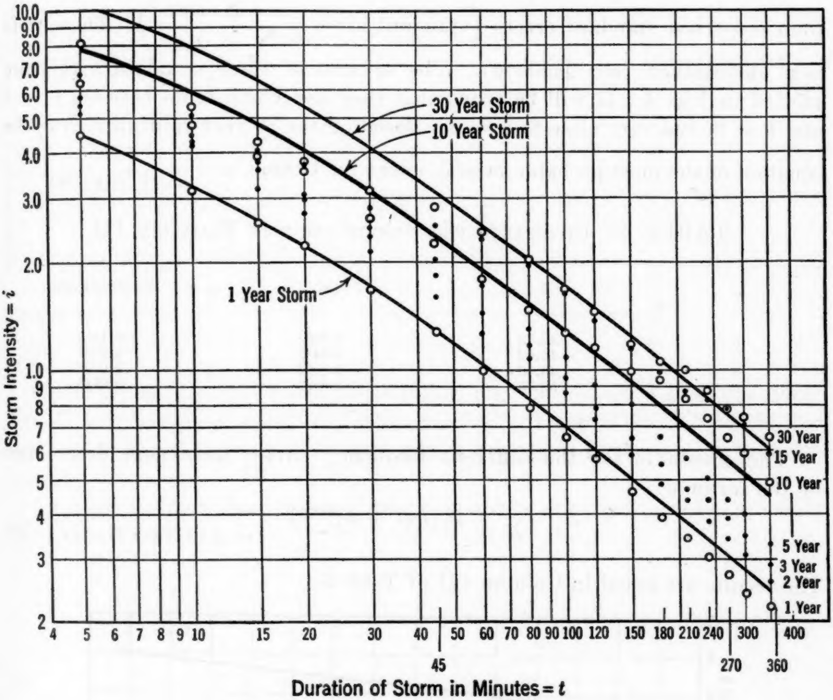

FIG. 3.—RAINFALL FREQUENCY CURVES.

In fitting a system of curves to these data, it is believed that the importance of each frequency curve should be weighted, according to the number of storms by which it was determined. The determination of the weighted average frequency curve was, therefore, the first step in the analysis.

In the logarithmic form, Equation (1)* (which is the general curve to be fitted) becomes $\log i = \log K + d \log (t + b)$. A value of 6 was first assumed for b, and K and d were determined by least squares. The normal equations for this solution are:

$$\log i = n \log K + d\, \Sigma\, [\log (t + 6)] \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

and,

$$\Sigma\, [\log i \times \log (t + 6)] = \Sigma\, [\log (t + 6)] \log K + d\, \Sigma\, [\log (t + 6)]^2 \quad \ldots (7)$$

Solving Equations (6) and (7) simultaneously, the resulting equation is:

$$i = \frac{40.73}{(t + 6)^{0.8368}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

* *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 720.

By the same method, Equation (1) was solved assuming, successively, $b = 7$; $b = 8$; and $b = 9$. In the resulting equations the corresponding values of K and d are as listed, in Table 4.

The relative worth of these equations is determined by finding the squares of their respective standard errors $\left(\text{standard error} = \sqrt{\frac{\Sigma \text{ errors}^2}{n}}\right)$. These were next determined (see Table 4). The squares of these standard errors are plotted in Fig. 4. It will be noted that they are a minimum between $b = 8$ and $b = 9$, and very close to $b = 8$. Based on the 30-year rainfall record, the equation of the most probable rainfall curve for Detroit is: $i = \frac{K}{(t + 8)^{0.855}}$.

TABLE 4.—CONSTANTS FOR SUBSTITUTION IN EQUATION (1).

b.	K.	d.	Standard error.
6	40.73	0.8368	0.1185
7	43.62	0.8482	0.0720
8	45.80	0.8554	0.0292
9	52.75	0.8822	0.0415

The constants for the different frequency curves were next determined by the formula:

$$K = \frac{\Sigma\,[i\,(t + 8)^{0.855}]}{n} \quad \text{.......................(9)}$$

The results are noted in Column (2) of Table 5.

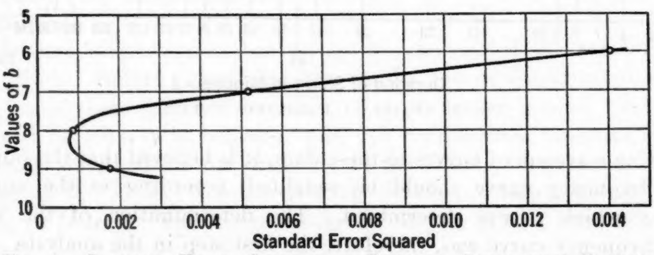

FIG. 4.—STANDARD ERROR, SQUARED, OF RAINFALL CURVE WITH VARIOUS VALUES OF b.

Fitting the curve, $K = a\,F^b$, to these data by least squares gives:

$$K = 37.6\,F^{0.263} \quad \text{.........................(10)}$$

The various K values for the different frequency curves have been computed and are noted in Column (3) of Table 5, for comparison with the original data. To summarize the results of this analysis, it would appear:

1.—That the use of the least squares method for analysis of the data is desirable.

2.—That the Detroit rainfall curve has a different rate of change from that at Boston, Mass., and, hence, this may be expected to be different in different localities.

3.—That the relations of the intensities of the different frequency storms are almost exactly the same for Boston and Detroit, and, hence, that this may be a universal law.

TABLE 5.—Frequency Constants for Detroit Rainfall Curves.

Frequency, in years.	Values of K ($= 37.6\ F^{0.268}$).		Percentage error of actual K.
	Actual.	Computed.	
(1)	(2)	(3)	(4)
1	35.9	37.6	4.5
2	45.5	45.1	0.9
3	51.9	50.2	3.2
5	56.8	57.4	0.9
10	70.3	69.0	1.9
15	80.9	76.7	5.1
30	86.1	92.1	6.5
Average	....	....	3.2

4.—That because this frequency relation may be universal it seems likely that, if rainfall data were available covering a more extended period, the empirical rainfall curves of Detroit would more closely fit the curves herewith derived.

5.—That the data from other districts should be analyzed in order to verify the Conclusions 1 to 4.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS

THE TRAINING WALL ACROSS THE LIAO BAR IN MANCHURIA

Discussion*

By B. Okazaki, M. Am. Soc. C. E.

B. Okazaki,† M. Am. Soc. C. E. (by letter).‡—After ten years of experimenting the Lower Liao River Conservancy Board has finally succeeded in obtaining about 3 ft. of additional depth in the channel by means of a single jetty across the Liao Bar. The results are praiseworthy. The writer agrees with the author as to the importance of utilizing the flow of the tides for their scouring effect§ especially since that flow is great enough when guided by a jetty. In the case of the Liao River, the scouring is due mostly to tidal currents rather than to the natural flow of the river. Across this bar, for example, the flow is about 210 000 cu. ft. per sec., which is more than ten times that of the fresh-water discharge from the stream itself.

Furthermore, unless the Upper Liao River is improved generally by the construction of dikes, levees, and other forms of bank protection there is not much hope of controlling the river flow as a scouring medium across the bar. The improvements that would be necessary should be constructed along the well-established principles of good engineering, contrasting in this respect with the present system of permitting the farmers who own the lands along the river banks to construct levees that are almost useless in times of great floods. At present, much of the river flow is naturally impounded on the low flat farm lands in the valley, due to ubiquitous breaks of the poorly located levees mentioned in case of flood. It is then released so gradually that any "flushing" effect that would be brought about by periods of maximum flow is considerably reduced. Under conditions as they exist at present it will require centuries for China to undertake these necessary improvements in the Upper Liao River.

* Discussion of the paper by P. N. Fawcett, M. Am. Soc. C. E., continued from October, 1930, *Proceedings*.

† Formerly Engr. in Chf. to Upper Liao River Conservancy Board, Newchwang, China.

‡ Received by the Secretary, August 21, 1930.

§ *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, pp. 930 and 942.

The present Conservancy Board does not have the necessary resources and, therefore, it must continue to depend upon tidal action for the proper maintenance of depth in the navigable channel. Before the relative importance of tidal flow as compared to river flow was generally recognized, members of the Liao River Conservancy Boards were generally of the opinion that all the flood water from the Upper Liao River should be reverted so as to pass the Port of Newchwang. Much of the river flow had previously been by-passed away from Newchwang through the Shuang-tai-tzu Channel. This branch broke through in 1890, eighty miles up stream from Newchwang, and empties into the Gulf of Liao-tung in the vicinity of Pan-shen-hsien (Fig. 10) many miles to the westward. Practically three-fourths of the entire discharge of the river is thus diverted away from Newchwang.

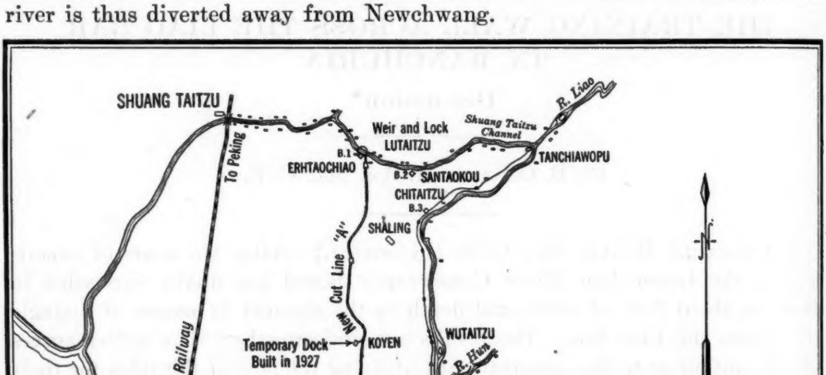

FIG. 10.—KEY PLAN OF LOWER LIAO BAR.

Handicapped by lack of funds, the Upper Liao River Conservancy Board was obliged to limit the extent of improvement works to the construction of a

movable weir across the Shuang-tai-tzu Channel at Erh-tao-chiao, fifteen miles below Tan-chia-wo-pu, although one lock was constructed in connection with this weir and a new canal about fourteen miles long was excavated from Erh-tao-chiao to Chia-shen-tsu. The cross-section of this canal was sufficient only to carry the ordinary low-water discharge of the Upper Liao River. However, with the excavated material built up as levees 400 ft. from the center line on each side, provision was made to meet the demands of part of the flood discharge to be expected.

The object of these improvements by the Upper Liao River Conservancy Board—working in co-operation with the Lower Liao River Conservancy Board as outlined by Mr. Fawcett and the writer—was (1) to recover the inland trade carried by junks; (2) to facilitate navigation in the vicinity of Newchwang in general; and (3) to supplement the scouring force of the tides across Liao Bar with the natural discharge of the river.

At the time these proposed improvements were begun, the writer began making several hydraulic investigations. He soon discovered that, despite the great extent of the drainage area contributing to the river flow, the maximum flood discharge at Tan-chia-wo-pu is only about 60 000 cu. ft. per sec. Comparing this with the 210 000 cu. ft. per sec. of tidal flow at the bar, the relative importance of these two sources of natural power may readily be seen. Subsequently, the Conservancy Board decided to focus its attention on the improvement of the most effective of these two forces rather than to expend the time and money necessary for improving a supply that would be relatively unimportant at best. It was decided that in order to be economical as well as successful from an engineering point of view the improvement of the fresh-water flow would have to await that of the entire river course on a large scale.

Aside from the foregoing considerations, there is another factor to be considered in any plan for utilizing the fresh-water flow. The Liao River may be classed as one of the muddiest rivers in the world. During 1925-26 the writer made extensive silt determinations at various places along the Liao River proper. Average determinations for each of the twelve months were made for surface, mid-depth, and bottom elevations. A summary of these results is presented in Table 1, in which the average silt content for the entire section is given at each station. By comparing the silt content of the main or Shwantaitzu River with that of the tributaries (Taitzuho and Hunho) combined, it is seen that the latter carries far less in suspension than the former. Therefore, it is clear that if more water is turned back into the Liao River proper the silt content of this stream would be considerably increased, thus transporting to the bar much more sediment than is being brought by the fresh-water flow and removed by the force of both tidal and fresh-water flow combined, as at present.

In other words, if the fresh-water flow is to be increased for its scouring effect on the bar, this should be done so as to exclude as much of the silt as possible. If an adequate supply of funds could be made available for this purpose it would be advisable to spend it in removing many of the sharp bends and irregularities in the lower reach of the river so as to increase the effect of

the tide and to shorten the navigable distance from the Gulf of Liao-Tung to old Newchwang (not new Newchwang where the present port is situated).

TABLE 1.—MONTHLY AVERAGE PERCENTAGE OF SILT, APRIL, 1925, TO MARCH, 1926, INCLUSIVE.

(Figures denote grammes of silt per 100 grammes of water.)

Station and river.	April.	May.	June.	July.	August.	September.	October.	November.	December.	January.	February.	March.	Monthly average.
Liao above Tangchiawopu	0.476	0.359	0.599	0.674	0.636	0.234	0.164	0.093	0.027	0.016	0.004	0.011	0.274
Liao below Tangchiawopu	0.497	0.371	0.614	0.646	0.623	0.262	0.166	0.108	0.017	0.014	0.004	0.013	0.278
Shuangtaitzu below Tangchiawopu	0.508	0.351	0.621	0.622	0.673	0.237	0.154	0.099	0.013	0.017	0.007	0.011	0.276
Liao above Sanchiaho	0.381	0.328	0.712	0.633	0.645	0.220	0.150	0.094	0.016	0.025	0.044	0.020	0.272
Liao below Sanchiaho	0.191	0.133	0.418	0.357	0.335	0.059	0.069	0.047	0.021	0.032	0.060	0.039	0.147
Taitzuho at Sanchiaho	0.085	0.042	0.251	0.238	0.226	0.026	0.032	0.034	0.008	0.015	0.028	0.016	0.083
Shuangtaitzu at Erhtaochiao	0.553	0.365	0.749	0.821	0.690	0.266	0.158	0.068	0.024	0.031	0.015	0.039	0.315
Liao at Chiashintzu	0.211	0.151	0.217	0.256	0.323	0.068	0.059	0.160	0.094	0.045	0.013	0.021	0.135

The original plan for the improvement of the Upper Liao River is practically completed and that for the Lower River, as described by Mr. Fawcett, will be completed in a few years, after obtaining the proper type of dredge for maintenance work. Then the trade of China should be promoted by taking advantage of the benefits derived from works of such magnitude and importance as those of the Upper and Lower Liao River Conservancy Districts. The immediate need is not so much to utilize the river flow as it is to appreciate fully the scouring effect of the tidal currents. This is the most important phase of the problem and should be accomplished with a view to maintaining a channel across the bar by means of dredges and the single training wall (as suggested by the author*). Furthermore, the tidal currents should be utilized, with river flow as it is at present; that is, without diverting any more muddy fresh water from up stream (although that may not be in conformity with the author's original idea).

It is desirable that the author submit a method of computing the quantity of silt deposits before selecting equipment for dredging and maintaining a 14-ft. channel alongside and beyond the end of the training wall and also to give estimate of what fraction of the entire job in view is capable of being done by the small dredge which was recently purchased by the Conservancy. Furthermore, the writer would like to suggest that Mr. Fawcett give the heights of the waves that occur in the vicinity of the bar. If there are no such data available perhaps the author could give some estimates based on his long experience. Is there any information available concerning the probable extent that storms and other sea disturbances would damage the bed of the channel after it has been dredged, especially near the end of the jetty?

* *Proceedings*, Am. Soc. C. E., May, 1930, Papers and Discussions, p. 942.

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PAPERS AND DISCUSSIONS
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HIGHWAYS AS ELEMENTS OF TRANSPORTATION

Discussion*

By Messrs. S. Johannesson, D. P. Krynine, Harold M. Lewis, and Donald M. Baker.

S. Johannesson,† M. Am. Soc. C. E. (by letter).‡—The author states that the highway considered in his paper will cost from \$3 000 000 to \$5 000 000 per mile.§ When the length of such highway is measured in miles rather than in feet, the total expenditure becomes so great that the first reaction is likely to be expressed in the question: Is it worth its cost? An affirmative answer is given in the paper, but it is of sufficient importance to warrant a re-statement.

The ultimate purpose of the highway is to decrease the cost of transportation of people and merchandise between the points served by it. This might also be expressed in other terms; for example, it might be said that the reason for building the highway is to relieve the congestion of existing traffic facilities or to decrease the time required to travel from one point to another. These statements are, in fact, only different ways of expressing the same definition.

It is probable that not less than 30 min. is saved in traveling over the new highway from the Holland Tunnel Plaza in Jersey City to the airport at Haynes Avenue, Newark. Using the data given in the paper, the traffic is estimated at 18 360 000 vehicles per annum‖ and is valued at 2 cents per car-min.¶ Therefore, the capitalized value of the savings in cost of transportation would be \$183 600 000, or, say, \$23 000 000 per mile of highway. It is apparent, therefore, that an expenditure of \$3 000 000 to \$5 000 000 is entirely justifiable, because it permits a saving several times the cost of construction.

* Discussion of the paper by Fred Lavis, M. Am. Soc. C. E., continued from October, 1930, *Proceedings*.

† Designing Engr., State Highway Comm., Jersey City, N. J.

‡ Received by the Secretary, October 1, 1930.

§ *Proceedings*, Am. Soc. C. E., August, 1930, Papers and Discussions, p. 1380.

‖ *Loc. cit.*, p. 1370.

¶ *Loc. cit.*, p. 1367.

The determination of the feasibility of a new highway should properly be based on an economic balance between the cost of producing the new facilities and the saving gained by them in cost of transportation. Generally speaking, this is directly proportional to the volume of traffic. Consequently, the amount of money that may be spent economically on new highway projects is directly proportional to the volume of traffic. When the traffic is heavy, as in the case considered by Mr. Lavis, this amount may often be found to be surprisingly large.

D. P. KRYNINE,* M. AM. SOC. C. E. (by letter).†—Modern technology and economics are intimately bound together; this statement is perhaps more true of Highway Engineering than of any other technical branch since the cost of a product moving from the producer toward the consumer, is often considerably increased by bad road conditions. Consequently, the necessity of an "economic design" arises, that is, a design in which the needs of transportation are met in the most efficient and, at the same time, the cheapest way.

Nowadays American engineers build roads in practically all parts of the world. Therefore, it is important for them to take into consideration the economic features of a given country or region in order to be able to design such roads effectively.

It may be observed that, in its economic development, a country or a region passes through the stages of being: (*a*) A colony or undeveloped country, such as America was about 1730; (*b*) a developed country, such as America in 1890 or 1900; and, (*c*) a congested area, such as New Jersey or New York at the present time (1930).

Each of these three steps may be found in different countries of this epoch. In the beginning of Period (*a*) there are practically no roads in a country or a region. Gradually, lanes or earth roads appear that satisfy the elementary needs of the settlers. The road system develops little by little, being adapted to the demands of growing industries. However, such a development of a road system involves technical errors, especially in properly selecting the right of way; and often subsequent generations are obliged to waste considerable sums of money in rectifying such errors.

Obviously, such a situation as that which existed about two centuries or more ago, when road experts had but poor ideas concerning highway technique and economics, cannot be tolerated to-day. A highway designer who deals with the road system in an undeveloped country, should prepare a comprehensive plan of construction. A thorough economic study of the region must precede any technical road survey; the existing volume of traffic should be determined, and the future volume should be estimated. Then, a proper type of each link of the system should be selected, almost in the same manner as is done by a structural engineer when determining cross-sections of a bridge according to the expected stresses. In highway design the stresses correspond to the volume of the traffic, and the cross-sections to the type of the road.

It must be borne in mind that in an undeveloped country or region the road system is a powerful means of controlling the development of industries,

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† Received by the Secretary, October 24, 1930.

and a foreign consulting engineer called to work in the design of such a road system, should have a series of conferences with Government officials and economists.

In an undeveloped country or colony the highways are indubitably elements of transportation. In time, however, railroads and other perfected means of transportation appear and, to them, highways cede their rôle of a means of through transportation. Thus, opens an epoch during which many (perhaps a majority) of roads and highways become merely municipal features like the water supply or the fire department.

The third stage is that so brilliantly described by Mr. Lavis. The construction of Route 25 in New Jersey is an introduction into the transportation system of a new kind of link that is something between "highway" and "railway." This new member of the transportation family may be called "super-highway" or "autostrade", as in Italy. Incidentally, Italy has constructed the following autostrades to date: (1) From Milan to Lakes Maggiore and Como; length, about 60 miles; width, 33 and 52 ft.; (2) from Rome to Ostia (a short distance); and, (3) from Naples to Pompeii (a short distance), and a vast program is under construction and consideration.*

HAROLD M. LEWIS,† M. AM. SOC. C. E. (by letter).‡—The author has described a modern highway as one which is designed with low grades and curvature and with a maximum elimination of delays from crossings of intersecting waterways, main highways, and local streets. Such is the "express highway" which has come to have an important place in the design for highway transportation in any large metropolitan area. New Jersey State Highway Route No. 25 was a pioneer project in such construction and has fully justified the faith of those who designed it.

Types of Express Highways.—That "express highways" are needed seems to be admissible without further discussion, but just what form such routes should take and what their relation should be to the other parts of the highway system and to the development of the adjoining land are questions requiring still further study. That there are various solutions of such problems is shown by examples already available. The "super-highways" radiating out from Detroit, Mich., on 204-ft. rights of way illustrate a type which combines the express highway with a main artery for rail transportation, facilities for local traffic, and the intensive development for business purposes of most of the fronting property. The West Side Elevated Highway under construction along the Hudson River water-front of Manhattan, in New York City, utilizes a portion of the air rights above the wide open space composed partly of public streets and partly of the marginal wharf or place under the jurisdiction of the Dock Department. The space beneath this highway will still be available for street traffic, except where ramps interfere. In no other place in Manhattan could such an elevated highway have been constructed without acquiring large areas, or paying damages to abutting buildings for taking away their light and air.

* See "Tabula viarum Romani Imperii," Rome, 1930.

† Cons. Engr.; Executive Engr., Regional Plan of New York and Its Environs, New York, N. Y.

‡ Received by the Secretary, October 31, 1930.

The City of Chicago has proposed an Avondale Super-highway, 10 miles long, from the heart of the city to its northwestern limits. This highway would parallel the tracks of the Chicago and North Western Railroad and would be elevated above the normal street system to which connections would be provided at intervals of about 1 mile, planned so that left-hand turns would be unnecessary. There would be an adjacent local low-level street, 50 ft. wide, for 9 of the 10 miles of the improvements.

The City of New York is planning a cross-town express highway in Manhattan to be constructed as an extension of a mid-town vehicular tunnel under the East River and to connect with a second vehicular tunnel under the Hudson River projected by the Port of New York Authority. This express highway will be a deep-level tunnel passing beneath all existing or projected rapid transit subways and will have no connections with the street system, except at the two ends—one near Second Avenue on the East Side and the other near Tenth Avenue on the West Side.

In Westchester County, New York, the County Park Commission has planned an express highway for general traffic to be constructed on a new right of way roughly parallel to the New York, New Haven, and Hartford Railroad. The right of way has been acquired, but to date construction has not been started.

In the Detroit District a 40-ft. elevated express toll highway has been proposed above the Detroit-Pontiac Line of the Grand Trunk Western Railway of the Canadian National System, as part of a $100 000 000 electrification and improvement program. The roadway would be 40 ft. above the rails, and intersecting highways would cross on an intermediate level between the railroad and highway. It is proposed to build the highway in sections and to establish the toll on zone rates.

The foregoing examples indicate the various forms which express highways are taking. In addition, there is the "parkway" which may provide an "express" highway restricted to passenger car traffic. Not all parkways may actually be express highways as some are primarily for local recreational purposes, but when—as in the case of the Bronx River, Saw Mill River, and Hutchinson River Parkways, in Westchester County, New York—they form main highways to or from metropolitan centers, they are closely related to such general traffic highways as those discussed herein.

Proposed Express Highway System for New York Region.—The Graphic Regional Plan prepared for New York and Its Environs included a comprehensive system of express highways, of which New Jersey State Highway Route No. 25, and the West Side Elevated Highway and the cross-town tunnel in Manhattan, form important parts. For smaller regions, or those with less geographical complications, it might not be necessary for the express highway system to cross the main business center, but only to skirt its edge. In this case, however, two lines were shown across the main center in Southern Manhattan. One of these utilized the Holland Tunnel and the other the projected mid-town crossings previously mentioned.

North and south routes were proposed on both the main water-fronts of Manhattan and also on both sides of the Hackensack Meadows, in New Jersey,

and a fifth route through Brooklyn and Queens Boroughs parallel to the East River. A total of six radial routes was shown, only one of which (the route described by Mr. Lavis) was extended to the edge of the Region. Route No. 25 was officially projected to Philadelphia, Pa. The other five routes extended out from 20 to 25 miles from the metropolitan center. Fig. 10 shows two additional express highways now proposed by the New Jersey State Highway Department. One of these is a branch of Route No. 25 through the center of Newark, the other would lead to the west across the congested areas of Essex County.

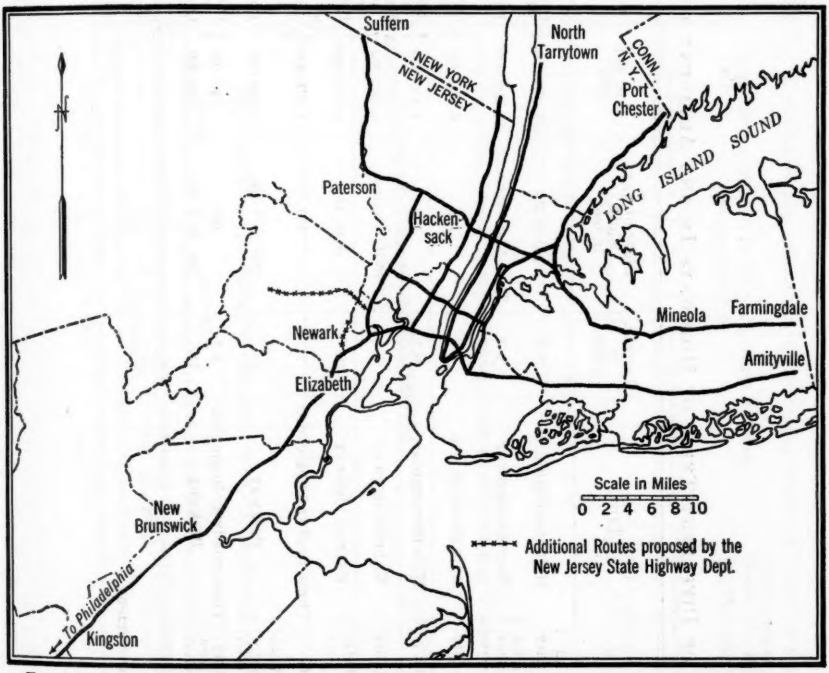

FIG. 10.—PROPOSED SYSTEM OF EXPRESS HIGHWAYS FOR NEW YORK AND ITS ENVIRONS.

Comparative Costs.—The cost of such a highway through intensively developed areas of high land values is tremendous, particularly if it becomes necessary to locate it entirely above or below the level of the existing street systems. In suburban territory, an express route may follow the surface of the ground throughout most of its length, and grade separations may be installed only at the main intersecting highways; secondary streets can terminate in part at the highway and in part cross by short offsets along the line of the express highway.

Many of the important links in the highway system may be amply supplied by wide boulevards or ordinary streets. The comparative costs of different types of streets are shown in Table 4 by figures for units of a mile in length.

It is evident from Table 4 that an express highway or wide parkway may be built through the less intensively developed sections of a city for only a fraction of the cost of providing new facilities of the same or even less capacity through

TABLE 4.—COMPARATIVE COSTS OF DIFFERENT TYPES OF HIGHWAYS IN AND ADJACENT TO NEW YORK CITY.

Name of highway.	Location.	Type.	Length, in miles.	Right-of-way width, in feet.	Cost per mile.	Remarks.
Sixth Avenue, Extension.	Manhattan, New York City	Main thoroughfare	0.6	100 to 210	$14 000 000†	Opened 1930; forms new approach to Holland Vehicular Tunnel.
Mid-town Tunnel..........	Crossing Manhattan, New York City, at or near 38th Street...............	Express highway	1.5	Two 2-lane tunnels	11 200 000*	Final plans being prepared.
Varick Street widening and Seventh Avenue, Extension...............	Manhattan, New York City	Main thoroughfare	1.3	100	6 300 000	Opened 1918; Varick Street widened from 65 to 100 ft.
Flatbush Avenue, Extension....................	Brooklyn, New York City.	Main thoroughfare	0.67	100	4 450 000	Approach to Manhattan Bridge; opened 1909.
West Side Elevated Highway.....................	Manhattan, New York City	Express highway	1.5	70-ft. viaduct	4 300 000	Section, Canal to 23d Street only; opened 1930; no right-of-way costs required.
New Jersey State Highway No. 25..............	Jersey City to Elizabeth, N. J......................	Express highway	13.	66 to 120	2 200 000*	Partly completed; remainder under construction.
Nassau Boulevard........	Queens, New York City...	Three-roadway boulevard	2.1	160	1 171 000‡	Section, Strongs Causeway to Fresh Meadow Road; includes paving of only a 30-ft. roadway.
Bronx River Parkway....	New York City and Westchester County, New York....................	Parkway	16.	200 to 1 200	1 060 000	Final section opened in 1925.
Kings Highway...........	Brooklyn, New York City.	Three-roadway boulevard	4.1	160	740 000	Opened 1926; includes paving of central roadway only.
Hutchinson River Parkway.....................	Westchester County, New York....................	Parkway	11.	200 to 1 000	370 000	Opened 1928.

* Preliminary estimates.
† Acquisition cost plus estimated cost of improvement.
‡ Estimate based on incomplete figures.

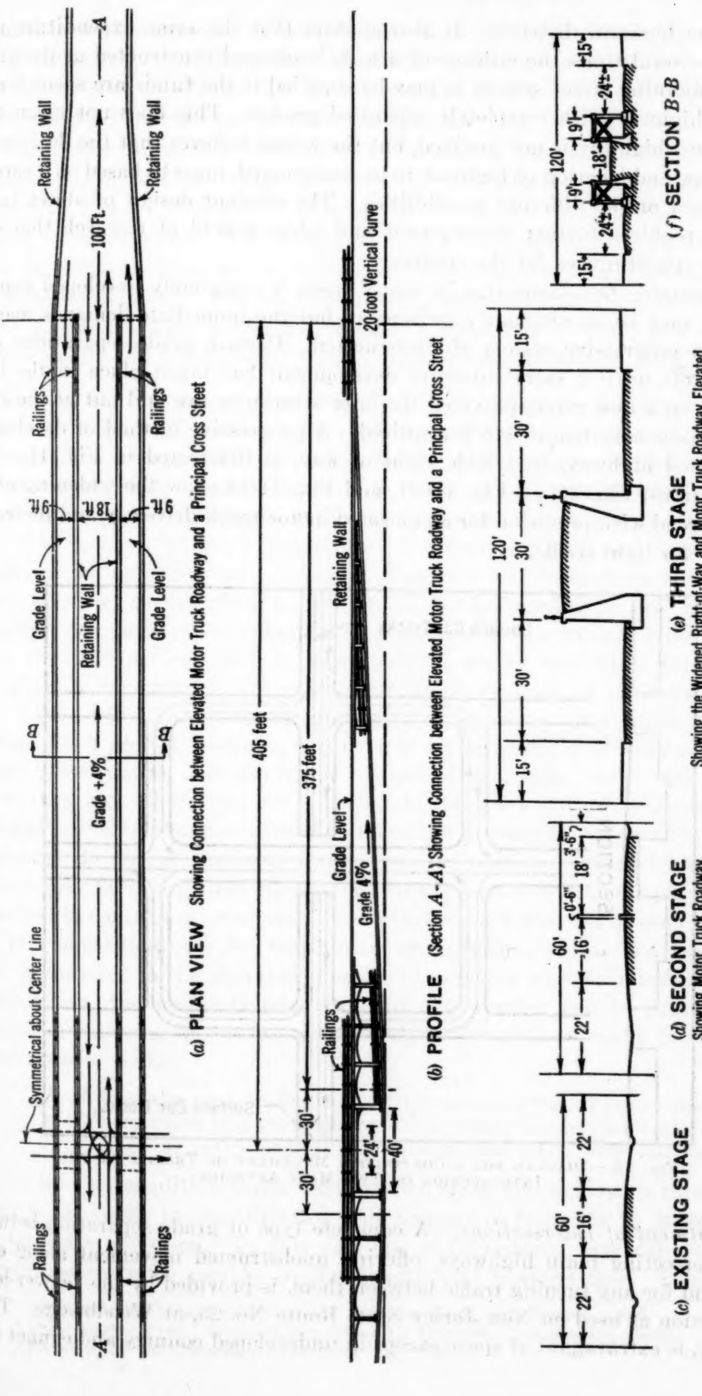

FIG. 11.—SKETCHES SHOWING POSSIBLE STAGES OF DEVELOPMENT OF MOTOR-TRUCK HIGHWAYS.

important business districts. It also appears that the same expenditure may provide several times the mileage of a wide boulevard constructed at the grade of the adjoining street system as may be supplied if the funds are spent for an express highway with completely separated grades. This does not mean that the express highway is not justified, but the writer believes that the decision as to the type and location of highway to be constructed, must be based on a careful comparison of the different possibilities. The efficient design of street intersections requires further development and offers a field of research that still presents opportunities for the engineer.

Progressive Development.—In many cases, a completely developed express highway may be an ultimate requirement, but the immediate demands may be met by a progressive system of construction. Certain grade separations may well be left until a more intensive development has taken place in the land fronting on a new route, provided the later structures are laid out at the start and the necessary real estate is acquired. A progressive method of developing an elevated highway, in a wide right of way, is illustrated in Fig. 11. The road sections, Fig. 11(*c*), Fig. 11(*d*), and Fig. 11(*e*) show the widening of an existing road with provision for an elevated motor-truck driveway and increased capacity for light traffic.

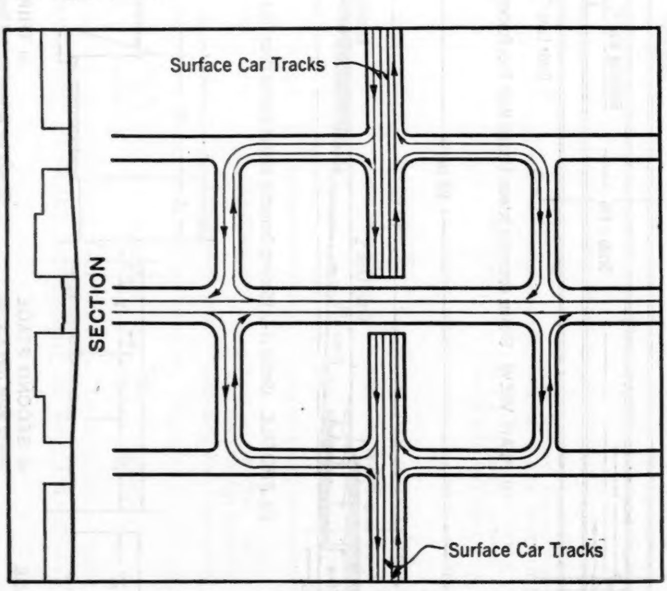

FIG. 12.—DIAGRAM FOR A CONTINUOUS MOVEMENT OF TRAFFIC AT THE INTERSECTION OF TWO MAIN ARTERIES.

Treatment of Intersections.—A complete type of grade separation between two intersecting main highways, offering unobstructed movement along each route and for any turning traffic between them, is provided by the "clover-leaf" intersection as used on New Jersey State Route No. 25, at Woodbridge. This, however, is extravagant of space except in undeveloped country and cannot well

be used in business areas. A single grade separation and the use of adjoining streets, however, may approximate the same results in a built-up area. A suggestion for such a solution has been developed by the Regional Plan of New York and Its Environs and is shown in Fig. 12.

The traffic circle is a solution of intersections that may be the only one feasible where several radiating routes of comparatively equal importance intersect. For an intersection of only two routes the writer feels that it will seldom be satisfactory. The substitution of a rectangular area for the circle, with the long axis parallel to the major of two intersecting highways, offers a far better solution. Studies for such intersections have been made by Mr. Fritz Malcher.* They might well provide a temporary solution at some intersection points and a permanent one at points of only secondary importance.

DONALD M. BAKER,† M. AM. SOC. C. E. (by letter).‡—Highway planning, like other phases of planning, is rapidly approaching a more or less exact science. The economic features, while not yet sufficiently evaluated to apply to all cases, are still adequate enough to make such planning more than an approximate guess. While the author himself states that the method of attacking this problem is not original, he is, however, to be complimented for applying the approach to the design of such a structure from an economic point of view, as described in his paper. It is hoped that this will be an incentive for more experimental work to determine the values of the different elements entering into such a project, which can be applied to future cases.

There is no question but that super-highways, such as that described by Mr. Lavis, will be built in the future in a number of the larger American cities. Each project, however, will have to be developed only after a thorough study of economics, and also of the method of financing. Such highways, far more than any other type, are a benefit to the users thereof, although in the general financing plan care should be taken not to overlook the fact that if these highways do relieve the local street systems, residents served by such local streets may be saved large future improvement costs and could reasonably be expected to contribute some small percentage toward the cost of such highways.

It is noted that only five traffic lanes are provided on this highway. While this is contrary to the standard practice of creating an even number of lanes, nevertheless, the writer believes that an odd number can be provided with considerable saving in construction and right-of-way costs, under the following conditions of traffic:

1.—Where the peak traffic in both directions is not of equal density.
2.—Where there is a great degree of diversity in the character of traffic, such as many high-speed vehicles and many slow-moving trucks.

In the first condition traffic studies often show that during morning or evening peaks traffic in one direction will amount to from 150 to 200% of that in the other direction. In this case the odd-numbered lane can be used to accommodate traffic moving in the direction of heaviest flow. If a reversal

* *The American City*, September and October, 1929, and August, 1930.
† Cons. Engr.; Pres., Board of City Planning Commrs., Los Angeles, Calif.
‡ Received by the Secretary, November 11, 1930.

of this occurs at another time of day, the odd-numbered lane may be used for the traffic in the other direction. A little education is sufficient to adjust the public to this situation and under these conditions an even number of lanes may normally represent useless expenditure.

Under the second condition in which, say, three or five lanes are provided (particularly three lanes), heavy slow-moving vehicles must be passed by faster and lighter vehicles and the existence of an extra lane used only for such passage will greatly expedite the traffic.

This all points to the fact that, before right-of-way and paving widths are definitely determined, a careful study of existing and prospective traffic should be made, in order to find out whether the extra lane necessary to furnish an odd number of trafficways is, or will be, necessary.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS AND DISCUSSIONS
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TESTS OF BROAD-CRESTED WEIRS
Discussion*

By Messrs. Sherman M. Woodward and David L. Yarnell

Sherman M. Woodward,† M. Am. Soc. C. E. (by letter).‡—Whenever flow is near the critical depth and critical velocity, the depth is in a condition of peculiar unsteadiness, leading easily to the formation of standing waves by some very slight disturbing factor. On this account, it will probably be difficult to devise any arrangement of the channel which will insure that at some prescribed point the depth shall be exactly the critical depth. This state of uncertainty does not extend, however, to the quantity of flow, since for a given total head it is at a maximum value during critical flow, and, therefore, independent of slight changes in depth. The result is that the very same conditions which are most unfavorable for an experimental measurement of critical depth are most favorable for the prediction of quantity of flow.

Compared with many other hydraulic phenomena, the flow over weirs is nearer stream-line flow and is, therefore, relatively more stable and less subject to accidental and erratic disturbance. Through a careful exploration of the velocity distribution throughout the sheet flowing over the weir crest it may be expected that the effect of each separate factor influencing the flow can be determined. After such an analysis it may be possible to know how the flow over the weir is influenced by the shape and angles of the approach channel, by the dimensions and curvature of the crest, by the friction on the crest, and by the nature of the exit conditions from the crest. The phenomena are particularly well adapted to study by the laws of similarity.

The ideal should be to accumulate experimental data sufficiently comprehensive and precise so that the exact flow over any full-sized overflow weir can be predicted in advance of its construction. The author and Professor Webb§ have made a most commendable contribution toward the realization of this ideal.

* Discussion of the paper by James G. Woodburn, Assoc. M. Am. Soc. C. E., continued from October, 1930, *Proceedings.*

† Prof. of Mechanics and Hydraulics, State Univ. of Iowa, Iowa City, Iowa.

‡ Received by the Secretary, November 11, 1930.

§ *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1604 *et seq.*

The paper opens up a series of fascinating questions, and it is to be hoped that it will be followed by further studies. This field appears to be particularly favorable for intensive studies designed to show the fundamental nature of stream flow.

DAVID L. YARNELL,* M. AM. SOC. C. E. (by letter).†—This paper presents interesting data on various types of broad-crested weirs both in regard to coefficients of discharge and the location of the critical depth.

During 1929 the U. S. Bureau of Public Roads, in co-operation with the State University of Iowa, conducted a series of experiments on the flow of water over full-sized sections of railway and highway embankments for the purpose of developing formulas for use in calculating the quantity of flood water flowing over such embankments.

Full-sized sections of single and double-track railway embankments, 10 ft. in length, were built in the laboratory testing canal. With the rails removed the embankments resembled somewhat the weir for Test Series *B* to *G*, Fig. 4.‡

The single-track railway test embankment with rails removed is shown in Fig. 13 (*a*). Profiles of the water surface are shown for free flow for quantities of from 12 to 78 cu. ft. per sec. The position of critical depth (indicated by circles) has been computed for each quantity of flow. It will be noted that, with certain exceptions, the locus of critical depth is approximately a vertical line. Fig. 13 (*b*) shows a check series of tests run on the same embankment for quantities of flow of from 15 to 74 cu. ft. per sec. The locus of critical depth was again found to be practically a vertical, as is also the case in Fig. 13 (*c*) which shows a series of tests on a double-track railway embankment (with rails removed) for quantities of flow of from 11 to 74 cu. ft. per sec.

The water-surface profiles were obtained from a series of staff gauges placed along both walls of the testing canal. Due to the continual pulsation of the water surface at the various gauges, the surface profile may be a few hundredths of a foot in error. This error is of great importance in locating exactly the position of critical depth. Hence, it is believed that for Fig. 13, all the loci of critical depths in each case would lie in verticals if the surface profiles could have been obtained more accurately.

Attention is called to the position of the tail-water for a flow of 74.50 cu. ft. per sec. in Fig. 13 (*b*). For this experiment the hydraulic jump occurred at Station 41, while the point of critical depth lies practically in the same vertical as in the case of the tests with free flow.

In connection with some tests on the hydraulic jump on sloping floors the writer had an opportunity to make a series of experiments on a weir with aprons inclined up stream and down stream. These experiments were made in a glass-walled channel, 2.5 ft. wide. The quantities of flow ranged from 3 to 8 cu. ft. per sec. The surface profiles and the loci of critical depths are shown in Fig. 14. In this case the locus of critical depth is in a vertical approximately at the crest of the weir.

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* Senior Drainage Engr., Bureau of Public Roads, U. S. Dept. of Agriculture, Iowa City, Iowa.

† Received by the Secretary, November 10, 1930.

‡ *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1598.

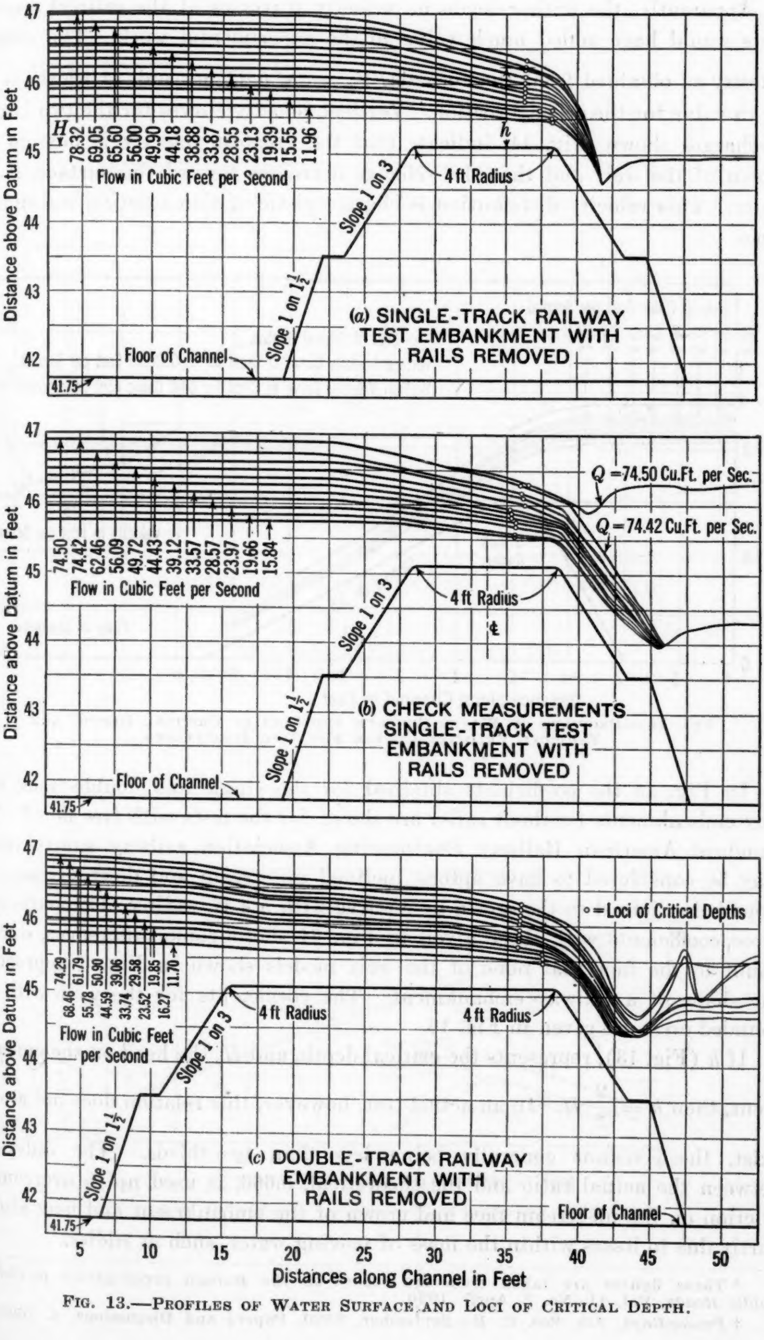

FIG. 13.—PROFILES OF WATER SURFACE AND LOCI OF CRITICAL DEPTH.

Apparently, the author made no velocity traverses at the critical section. This would have added much value to the experimental work. The critical velocity as obtained from the formula, $V_c = \sqrt[3]{g\,Q}$ per unit of width, is the mean value for the critical section. Vertical velocity curves for the two largest discharges shown, Fig. 14, indicate that the greatest velocity is next to the crown of the weir and that the velocity decreases toward the surface of the water. This velocity distribution is characteristic of that existing in bends of pipes.

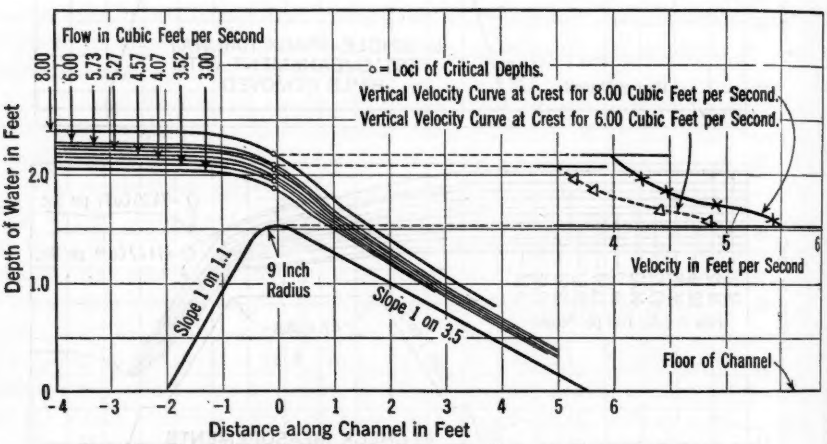

FIG. 14.—PROFILES OF WATER SURFACE AND LOCI OF CRITICAL DEPTH, AND VERTICAL VELOCITY CURVES FOR TWO DISCHARGES.

In Fig. 15 the coefficients obtained for the single and double-track railway embankments (without rails) are shown for the tests with free flow.* The standard American Railway Engineering Association railway embankments may be considered to have aprons inclined up stream and down stream, the slope being joined to the crown by a curve with a 4-ft. radius. Comparison of these coefficients with those given in Fig. 6† shows some differences, due no doubt to the fact that none of the weir models shown in Fig. 6 approaches the shape of a railway embankment. The coefficients for the weir with the rounded crest are given in Fig. 16.

If h (Fig. 13), represents the critical depth, and H, the head on the embankment, then $h = \frac{2}{3} H$. In an actual test, however, this relation does not always exist, the constant generally being less than two-thirds. The difference between the actual ratio and the theoretical, 0.666, is used up in overcoming friction on the up-stream face and crown of the embankment and may also be partly due to losses within the mass of moving water, such as eddies.

* These figures are taken from the report on the Bureau investigation printed in *Public Roads*, Vol. 11, No. 2, April, 1930.

† *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1602.

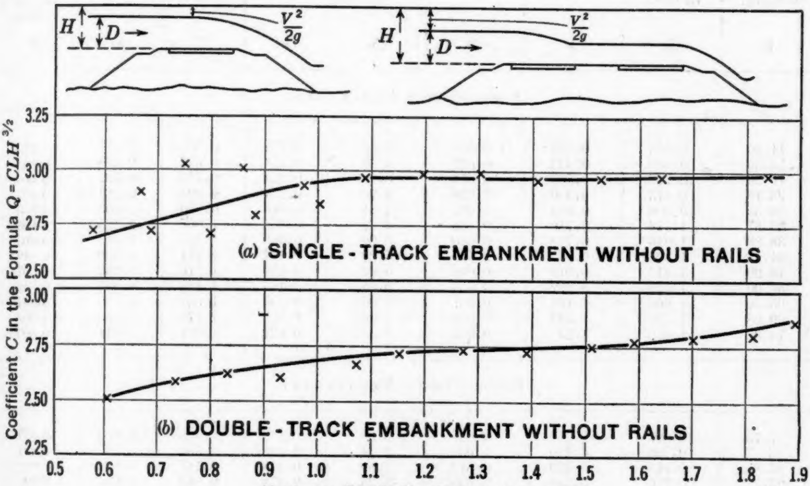

FIG. 15.—CURVE SHOWING VARIATION OF COEFFICIENT WITH HEAD.

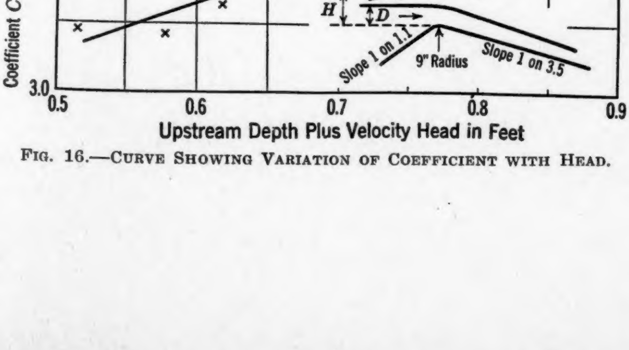

FIG. 16.—CURVE SHOWING VARIATION OF COEFFICIENT WITH HEAD.

TABLE 7.—COMPUTATION OF FRICTION LOSSES.

Discharge, in cubic feet per second. (1)	Head on embankment, H, in feet. (2)	Critical depth, h, in feet. (3)	Ratio, $\frac{h}{H}$. (4)	Critical velocity, v_c, in feet per second. (5)	$\frac{V_c^2}{2g}$. (6)	$h+\frac{V_c^2}{2g}$. (7)	$H+\frac{V_1^2}{2g}$. (8)	Loss due to friction, Column (8) minus Column (7), in feet. (9)
SINGLE-TRACK EMBANKMENT.								
11.96	0.576	0.355	0.616	3.37	0.177	0.532	0.577	0.045
15.55	0.686	0.422	0.615	3.68	0.211	0.633	0.688	0.055
19.39	0.796	0.489	0.614	3.96	0.244	0.733	0.799	0.066
23.13	0.876	0.550	0.628	4.20	0.274	0.824	0.881	0.057
28.55	0.996	0.634	0.637	4.51	0.316	0.950	1.003	0.053
33.87	1.116	0.710	0.636	4.78	0.355	1.065	1.125	0.060
38.88	1.216	0.778	0.640	5.00	0.389	1.167	1.227	0.060
44.18	1.306	0.847	0.648	5.22	0.424	1.271	1.320	0.049
49.90	1.416	0.918	0.648	5.43	0.458	1.376	1.433	0.057
56.00	1.516	0.992	0.654	5.65	0.496	1.488	1.537	0.049
65.60	1.666	1.101	0.661	5.95	0.550	1.651	1.692	0.041
69.05	1.736	1.142	0.658	6.06	0.571	1.713	1.765	0.052
78.32	1.876	1.24	0.661	6.32	0.621	1.861	1.911	0.050
DOUBLE-TRACK EMBANKMENT.								
11.70	0.596	0.350	0.587	3.35	0.175	0.525	0.602	0.077
16.27	0.726	0.436	0.601	3.74	0.218	0.654	0.735	0.081
19.85	0.816	0.499	0.612	3.99	0.248	0.747	0.829	0.082
23.52	0.916	0.558	0.609	4.23	0.278	0.836	0.933	0.097
29.58	1.046	0.648	0.619	4.56	0.323	0.971	1.069	0.098
33.74	1.126	0.708	0.629	4.76	0.352	1.060	1.155	0.095
39.06	1.236	0.781	0.632	5.01	0.390	1.171	1.271	0.100
44.59	1.346	0.854	0.634	5.23	0.425	1.279	1.388	0.109
50.90	1.456	0.931	0.639	5.46	0.464	1.395	1.507	0.112
55.78	1.536	0.991	0.645	5.64	0.495	1.486	1.594	0.108
61.79	1.616	1.059	0.655	5.83	0.528	1.587	1.683	0.096
68.46	1.736	1.135	0.654	6.04	0.567	1.702	1.812	0.110
74.29	1.806	1.194	0.662	6.21	0.600	1.794	1.892	0.098

In Table 7 the actual ratios for the single-track (Fig. 13 (*a*)) and double-track (Fig. 13 (*c*)) embankments are given as well as the amount used up in friction, etc. A comparison of the two sets of data shows that a little greater loss occurs, on the average, in the double-track than in the single-track embankment. This is to be expected since the double-track embankment is the wider.

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AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

DECEMBER, 1930

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BOARD OF DIRECTION MEETINGS

January 19-20, 1931:

A Quarterly Meeting will be held at New York, N. Y.

ANNUAL MEETING, NEW YORK, N. Y.,
January 21, 22, and 23, 1931

January 21, 1931:

Morning.—Annual Meeting. Conferring of Honorary Membership, and Presentation of Medals and Prizes.

Afternoon.—Symposium on Value to Engineers of Work of Government Bureaus.

Evening.—President's and Honorary Members' Reception and Dinner Dance.

January 22, 1931:

Morning.—Technical Division Sessions.

Afternoon.—Technical Division Sessions.

Evening.—Entertainment and Smoker.

January 23, 1931:

All-Day Excursion.

SPRING MEETING, NORFOLK, VA.,
April 15-17, 1931

The Reading Room of the Society is open from 9:00 A. M. to 5:00 P. M. every day, except Sundays, New Year's Day, Washington's Birthday, Memorial Day, Fourth of July, Labor Day, Thanksgiving Day, and Christmas Day; during July and August, it is closed on Saturdays at 12:00 M.

Members, particularly those from out of town, are cordially invited to use this room on their visits to New York, to have their mail addressed there, and to utilize it as a place for meeting others. There is a file of 291 current periodicals, the latest technical books, and the room is well supplied with writing tables.